Proceedings of the First Annual Meeting of the Georgia Association of Mathematics Teacher Educators

October 17, 2007
Rock Eagle Center
Eatonton, Georgia

Georgia Association of Mathematics Teacher Educators (GAMTE)

GAMTE 2007 Proceedings Committee

Submissions for GAMTE's Annual Conference Proceedings are peer-reviewed by the Proceedings Committee.

Purposes and Goals of GAMTE

The purpose of GAMTE is to encourage and facilitate the improvement of mathematics teacher education across the state of Georgia. The goals of the organization are to: facilitate communication and collaboration among mathematics teacher educators between and within all educational levels; coordinate activities and work collaboratively with other associations, organizations, and governmental (national, state, and local) units to strengthen the mathematical, pedagogical, and clinical preparation of mathematics teachers at all levels (P-college); facilitate collaboration among mathematics teacher educators who are members of different academic units, such as departments of mathematics and departments of education; promote leadership among mathematics teacher educators in the broader mathematics education community; encourage research related to mathematics teacher education, especially which identifies factors that contribute to improving the preparation and professional development of mathematics teachers at all levels; encourage and organize programs and meetings focusing in issues related to the preparation and professional development of mathematics teachers; and foster the incorporation of appropriate technology into teacher education programs and professional development opportunities in mathematics at all levels (P - college).

Special thanks to the Department of Early Childhood Education,
Georgia State University for preparing copies of the proceedings

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Preservice Teachers' Mapping Structures Acting on Representational Quantities

Günhan Caglayan, University of Georgia, sezen@uga.edu

Abstract: *In this article, I write about my research on five preservice secondary teachers' (PST) understanding and sense making of representational quantities associated with magnetic color cubes and tiles. Data came from individual interviews during which I asked PST problems guided by five main tasks: prime and composite numbers, summation of counting numbers, odd numbers, even numbers, and polynomial expressions in x and y. My work drew upon an analysis framework (Behr et. al, 1994) supported by a unit coordination construct (Steffe, 1988) associated with linear and areal quantities inherent in the nature of figures produced by these PST. Linear quantities can be thought of as generated via linear measurement units (e.g., inches, centimeters, units) whereas areal quantities are generated via areal measurement units (e.g., square inches, square centimeters, square units, etc.) I used thematic analysis supported by constant comparison and retrospective analysis to explain my theories and hypotheses concerning PST's representational quantities. I developed a data analysis framework which I named "Relational Notation" to describe these PST's understanding of linear and areal units. PST also treated the quantitative multiplication and addition operations as some kind of functions, mappings, when expressing the area of their growing rectangles made of magnetic color cubes and tiles as sums and products. Their behavior necessitated the existence of another component for my data analysis framework which I called "Mapping Structures"*

The Mathematical Preparation of Secondary School Teachers

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Abstract: *In the summer of 2007, a group of doctoral students at the University of Georgia gathered to discuss the mathematical preparation of secondary teachers. The group used Mathematics for High School Teachers: An Advanced Perspective by Usiskin, Peressini, Marchisotto, and Stanley (2003) as the catalyst for the discussion. Participants agreed that future teachers need opportunities to examine high school and college mathematics differently from the way they had as students, with specific emphasis on connections, representations, and history. Features of this text that were highlighted in the discussions were the attention topics with commonly held misconceptions, the historical rationales and development of mathematical topics, and the role of mathematical definitions. Group members felt that, depending on one's purpose for using the text and the backgrounds of the prospective teachers, this text could be used, in conjunction with supplemental materials, in a variety of capacities: for a capstone course, a connections course, or a set of replacement mathematics courses.*

Assessing Understanding of Multiplication through Words, Pictures, and Numbers

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Abstract: *The objective of this session is to engage mathematics teacher educators in a discussion of how to assess an understanding of the concept of multiplication as an operation and its relationships to other operations. The session will begin with a presentation of a previously published study assessing children's understanding of multiplication as grouping and the relationship between multiplication and addition. The assessment asked a series of problems involving words, pictures, and numbers. The results of the study indicate that the types of problems asked were successful in providing evidence of children's understanding of multiplication. The study also found that a group of third grade children had developed a better understanding of multiplication after just one multiplication unit from Investigations in Number, Data, and Space than a comparable group of fourth grade children had developed from an entire year of a traditional emphasis on memorizing multiplication facts. An interactive discussion of potential uses of this study and its assessment format in teacher education will follow the presentation.*

Using Technology to Design Teaching Modules in Mathematics and Science

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Abstract: *Technology is changing the way in which mathematics and science are taught, and this radical transformation in teaching is causing teachers to take a closer look at how lessons are designed. In an effort to demonstrate how to design instructional modules using technology, this paper will include the following: 1)A review of the National Educational Technology Standards for teachers to establish a framework for the development of the teaching modules; 2)instructional designs and techniques with special emphasis on multiple intelligence and critical thinking skills; 3) strategies and techniques for infusing technology into a standard based curriculum; and 4) an analysis of the evaluative data completed by pre-service teachers to determine the effectiveness of the modules. Middle and elementary pre-service teachers at a historically Black university are required to take a course, Integrated Methods of Teaching Science and Mathematics and the previously mentioned instructional modules were used in this course. The learning modules provided the pre-service teachers with technology enhanced experiences that were aligned with the national science and mathematics standards.*

Pre-service teachers were asked to complete five modules which included both formative and summative assessments. They were also asked to complete a survey to obtain their perceptions of the teaching strategies, their performance, the use of tools and assistance, and comfort with using the computer. The only statistically significant correlation ($p<.01$) was between performance and teaching strategies.

During the exit interview for the course, five of the pre-service teachers indicated that they liked the web-enhanced course and that it was good for their personal schedules; however, all the students had a concern about the required assignments and the amount of time required to complete them. All but one of the students indicated that they would take another web-enhanced course. Teaching modules that incorporate the use of technology are an excellent way to meet the learning needs of a diverse population

Change and Relationships in Elementary Preservice Teachers' Mathematics Pedagogical Beliefs, Teaching Efficacy Beliefs, and Content Knowledge

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Abstract: *This study investigated the mathematics beliefs and content knowledge of 103 elementary preservice teachers in a developmental teacher preparation program that included a two course mathematics methods sequence. Preservice teachers' pedagogical beliefs became more cognitively-oriented during the teacher preparation program with these changes occurring during the two methods courses. Pedagogical beliefs remained stable during student teaching. The preservice teachers also significantly increased their personal efficacy for teaching mathematics throughout the program with these shifts occurring across both methods courses and into student teaching. Pedagogical beliefs and teaching efficacy beliefs were not related at the beginning of the program, but, in general, were positively related throughout the program. In addition, the preservice teachers' pedagogical beliefs were positively related to their specialized content knowledge for teaching mathematics at the end of the program.*

The Kennesaw State University Mathematics Methods Model

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Abstract: *Kennesaw State University's comprehensive, nine-credit-hour, methods course integrates general and mathematics-specific pedagogical training with a structured four-week field experience prior to student teaching. This course blends essential units on conceptual understanding of mathematics, lesson planning, assessment, classroom management, and diversity with mathematics-specific methods. All topics are aligned with National Council of Teachers of Mathematics standards and Georgia Performance Standards. Throughout the course, students complete a variety of assignments that require them to practice the skills highlighted in class readings and discussions, and they adapt and generalize those skills during their field experiences. Students have numerous opportunities in class and in the field to implement and to reflect upon pedagogical and assessment strategies and to receive feedback from course instructors, from other students, and from collaborating teachers. This intense course has many benefits and challenges for both the instructors and the students, but it is one of the most highly anticipated courses of secondary mathematics education majors at Kennesaw State University. With strong support of department administrators and the entire mathematics education faculty, this methods class has been quite successful in preparing the seniors for their student teaching experiences.*

Preservice Teachers' Mapping Structures Acting on Representational Quantities[1]

Günhan Caglayan, University of Georgia, sezen@uga.edu

Abstract

In this article, I write about my research on five preservice secondary teachers' (PST) understanding and sense making of representational quantities associated with magnetic color cubes and tiles. Data came from individual interviews during which I asked PST problems guided by five main tasks: prime and composite numbers, summation of counting numbers, odd numbers, even numbers, and polynomial expressions in x and y. My work drew upon an analysis framework (Behr et. al, 1994) supported by a unit coordination construct (Steffe, 1988) associated with linear and areal quantities inherent in the nature of figures produced by these PST. Linear quantities can be thought of as generated via linear measurement units (e.g., inches, centimeters, units) whereas areal quantities are generated via areal measurement units (e.g., square inches, square centimeters, square units, etc.) I used thematic analysis supported by constant comparison and retrospective analysis to explain my theories and hypotheses concerning PST's representational quantities. I developed a data analysis framework which I named "Relational Notation" to describe these PST's understanding of linear and areal units. PST also treated the quantitative multiplication and addition operations as some kind of functions, mappings, when expressing the area of their growing rectangles made of magnetic color cubes and tiles as sums and products. Their behavior necessitated the existence of another component for my data analysis framework which I called "Mapping Structures"

[1] The author wishes to thank Bob Allen, Kathy Daymude, Sarah Donaldson, Ryan Fox, Gloria Jones, and Margaret Morgan for their thoughts, ideas, participation in the seminar, and ongoing conversations on this topic. I also wish to thank Jeremy Kilpatrick for his guidance and for sharing his personal experiences.

Preservice Teachers' Mapping Structures Acting on Representational Quantities

Mathematics gives rise to quantities which can be represented with physical objects mostly referred to as manipulatives. When representing these quantities, both students and their teachers should be proficient in identifying the characteristics of those quantities. Whole numbers can be thought of as made of units of 1. For instance, the number 12 can be thought of as a collection of twelve singleton units, or as four 3-units, or even as six 2-units. Moreover, in a representational situation, twelve little black square tiles can be used to model the representational quantity *12*. Odd integers can be represented as symmetric L-shaped figures, while even integers can be represented as rectangles with dimensions 2 by half the integer made of color tiles (Caglayan, 2006). Prime and composite numbers can be represented with various rectangular configurations made of color tiles as well (Caglayan, 2007). Color tiles serve to model not only positive integers, but also polynomial expressions in x and y.

Representation of irreducible quantities such as color tiles representing a "1", an "x", and a "y"; as well as larger representational quantities made of these sub-quantities is reminiscent of the "unitizing process." All the little pieces (e.g., each magnetic color cube denoting a "1" of a special number, each different size tile piece denoting a "1", an "x", a "y", an "x^2", an "xy", or a "y^2") and their various combinations (e.g., a 4 by 2 rectangle – made of 8 irreducible units of 1 – conceptualized as the unitizing of the even number 8, a $2x+y+1$ by $x+3y+2$ rectangle – made of 2 irreducible units of x^2, 3 irreducible units of y^2, 7 irreducible unit of xy, 5 irreducible units of x, 2 irreducible units of 1, 5 irreducible units of y – conceptualized as the unitizing of the polynomial expression $2x^2+7xy+3y^2+5x+5y+2$) serve for an essential theoretical construct which I define as Representational Unit Coordination.

Theoretical Framework

In its true nature, coordination is about "making various different things work effectively as a whole[2]." In the context of my study, it refers to the conception of unit structures in relation to smaller embedded sub-units within these unit structures, or, bigger units formed via iteration of these unit structures. In the multiplicative situation, for instance, the conception of 5 as 5 units of 1 is one way of coordinating units: 5 as a (composite) unit of 1. As another example, 35 can be

[2] Cambridge Advanced Learner's Dictionary online. Retrieved December 31, 2006 from http://dictionary.cambridge.org/define.asp?key=17018&dict=CALD

coordinated multiplicatively as 5 (composite) units of 7 (composite) units of 1. Unit coordination has been previously studied by various researchers in the mathematics education field. For example, Steffe analyzed the coordination of different levels of units in whole number multiplication problems which is reminiscent of a key concept in multiplication, i.e., the notion of composite units (Steffe, 1988). The essence of multiplication lies in fact in distributive rather than repeated additive aspect (Confrey & Lachance, 2000; Steffe, 1992). In the example above, the multiplication of 5 by 7 can be thought of as the injection of units of 7 (each being units of 1) into the 5 slots of 5, each slot representing a 1. In this example, the conceptualization of each singleton unit describing a unity, i.e., 1, stands for a *first level of unit coordination*. Moreover, 5 and 7 can be conceptualized (as composite units of 1) as 5×1 and 7×1, respectively1, as a *second level of unit coordination*. The product 5×7 which denotes 5 (composite) units of 7 (composite) units of 1, can be conceptualized as a *third level of unit coordination*. Some other researchers also studied unit coordination in a fractional situation (e.g. Lamon, 1994; Olive, 1999; Olive & Steffe, 2002). Additionally, work on intensive (e.g., miles per hour) and extensive quantities (e.g., number of hours) reflect unit coordination as well (Kaput, Schwartz, & Poholsky, 1985; Schwartz, 1988; Author (Olive & Caglayan, 2007) work on quantitative unit coordination and conservation also takes the unit coordination issue into account.

Representational Unit Coordination can be defined as the different ways of categorizing units arising from the modeling of identities on representational quantities as area as a product and area as a sum of the corresponding special rectangles made of magnetic color cubes and tiles. In my study, preservice teachers started with the area as a product concept. In its most basic sense, e.g., area of a rectangle, is defined as the product of its two dimensions. I am talking about the area of a rectangle, and not any other geometric figure, because the identities of representational quantities students analyzed via magnetic color cubes or tiles were always about a rectangle – prime rectangle, composite rectangle, odd rectangle, even rectangle, addition of odd and even integers generated as a growing rectangle, and polynomial rectangle. Coordination of these two dimensions, i.e., the arrangement of these two linear units in a particular order as an ordered pair such as (a, b) or (b, a), defines the first part of my construct RUC: *Multiplicative Type RUC*. The analysis of the other important concept, area as a sum (of a special number rectangle), is prone to several, not necessarily hierarchical levels of RUC. *Additive Type RUC* stands for the coordination, the arrangement of (in general two or more) areal units as n-tuples such as [2, 2, 2] or [3, 3] for the composite rectangle of 6. For this RUC type, areal units being coordinated have something in common. For instance, for the composite rectangle of 6, the 2's in [2, 2, 2] are interesting because 2 is a factor of 6, which is why this special additive type RUC is called *Equal Addends Type RUC*. Moreover, the coordination of less interesting (irreducible) areal units (of 1) as n-tuples such as [1, 1, 1, 1, 1, 1] for the same example, composite rectangle of the special number 6, necessitates the existence of another additive type RUC which is called *Irreducible Addends Type RUC.* There are actually ten additive type RUC's.

Context and Methodology

I conducted my study with PST's enrolled in the Mathematics Education Program of a university in the southeastern United States. I interviewed 5 PST's individually twice during January and February 2007. Duration of each session was about 60-75 minutes and each interview session was videotaped. The focus was on problems on identities for prime and composite numbers along with summation of counting numbers, odd and even integers as well as products and factors of polynomials modeled with magnetic color cubes and tiles. I selected my participants from two different undergraduate level mathematics education classes. Ben, Stacy and John came from the "Concepts in Secondary School Mathematics" class of 11 enrolled preservice teachers while Nicole and Ron came from the "Teaching Geometry and Measurement in the Middle School" class of 22 enrolled preservice teachers. All these five students volunteered to participate in my study. All names in this study are pseudonyms.

I used thematic analysis supported by constant comparison of the interviews and retrospective analysis. I also simplified and extended the *generalized notation for mathematics of a quantity* (Behr et al., 1994) in such a way as to cover identities that equate summation and product expressions of special numbers.

Results

In the context of prime and composite numbers, I asked the PST to represent a composite number (e.g., 15 and 28) and a prime number (e.g., 5 or 7) using wooden cubes. As for the summation of counting/odd/even numbers, the common direction for all the interview students was to represent counting/odd/even numbers using different magnetic color cubes for each number and add them so that they generate a growing rectangle. In the context of polynomials in x and y, they were asked to represent products and factors of polynomial expressions using different sized color tiles.

Multiplicative Representational Unit Coordination (MRUC)

Multiplicative type RUC arose from various usages by the PST such as "It [areal 6] is [linear]

6 and [linear] 1", "This [linear x] and this [linear 1] to find this [areal x]", "2 by one half of the number", "When you put this length [linear 1] and that length [linear 1] together", "This edge [linear 1] right here and this edge [linear 1] right here", "This edge [linear x] by this [linear x] edge". For all such usages, I used a relational notation of the form (a,b) where a and b stand for the corresponding linear quantities represented by the dimension tiles.

In the "Summation of Counting Numbers" activity, all PSTs came up with a similar growing rectangle pattern as in figure 1. When I invited Nicole to discuss the subunits corresponding to odd and even numbers on her growing rectangle, Nicole said that the odd numbers are all represented with straight lines. MRUC can not be concluded from her usage "straight lines" because a line is of linear nature and one can not have a relational notation of ordered pair representing a linear quantity. As for the even number subunits, however, she said that they all have length greater than 1, which lead me to think that she was beginning to think in a multiplicative way, as reflected in the following protocol.

Figure 1. Growing Rectangle Sequence Based on Rectangular Subunits.

Protocol 1: Nicole's Multiplicative RUC of Even Number Subunits.

I: What is common about the even numbers?

N: They are all by 2. Because in all even numbers 2 is a divisor or factor.

I: How would you describe the area of this big rectangle as a product?

N: I would say three times seven. Three inches times seven inches would give me 21 inches squared.

From this protocol, I deduce that the area of the growing rectangle, in the context of multiplicative RUC, can be expressed via the relational notation of ordered pair (3,7) of linear units, as described by Nicole's usage "three times seven". Her last usage "three inches times seven inches would give me 21 inches squared" calls for an operational type terminology "Mapping Structures" which will be described below. Nicole was able to see the multiplicative nature of the area of the growing rectangle for the special case corresponding to the 6^{th} stage (figure 1), however, she could not generalize this for any growing rectangle of the sequence. As for the subunits, she specified only one of the dimensions, as can be inferred from her usage "They are all by 2. Because in all even numbers 2 is a divisor or factor". This usage calls for a relational notation of the form (2,·) where the dot "·" represents the missing unspecified linear unit with value "half the even number". Ben was not able to generalize the linear unit corresponding to "any even number", either.

When I asked Ron about the pattern generating his growing rectangle for the addition of counting numbers activity, he said that every time you get an odd number, you can put it below the growing rectangle. The following protocol illustrates this point.

Protocol 2:

Ron's Description of the Growing Rectangle Sequence:

Bridge Connection between Consecutive Subunits.

R: The next even number will add two rows to what... see when we had three [meaning when he added the odd integer three] it was three by two [meaning the growing rectangle]. So you add two by two which is four, you get two more rows [meaning, two more cubes right next to the odd integer three] and it makes it [inaudible] to get a five. Now you have five, and for the next odd number you are gonna need a seven, which is why you add a 2 by 3 rows [meaning the two extra cubes will come from the even number 6].

Ron realizes that each even number subunit of the sequence serves as a bridge that connects the two consecutive odd number subunits. I can also infer that, just by looking at his growing rectangle sequence (similar to figure 1) that Ron knows that the difference between any two consecutive odd integers is 2. And "that 2" in fact is provided by the even integer subunit which is placed right in between the consecutive odd integers. Ron was the only student to make use of this strategy, which I name *Bridge Connection between Consecutive Subunits*. In Thompson's (1988) words, "To reason quantitatively is to reason about quantities, their magnitudes, and their relationships with other quantities" (p. 164) . Ron's *Bridge Connection between Consecutive Subunits* strategy has a strong indication of the quantitative reasoning described by Thompson. Ron's subunits are not only of multiplicative (and additive) nature – hence can be described via a multiplicative (and additive) type relational notation – but Ron is also very explicit in how these quantities exist on their own as well as in relation to their neighbors, which serve as a bridge at each step. And it is because of these bridges that all subunits and the growing rectangle made of those subunits come to exist as quantities for Ron.

I then asked Ron what is common about odd numbers and their area as a product because I wanted him to make a generalization for the odd numbers. He said that all the odd numbers can be represented by a rectangle whose dimensions are 1 by the odd number itself, at the same time pointing to their rectangles. Ron's generalization about odd number subunits could be expressed as a relational notation of ordered pair (1,n) of linear units where n represents the value of the corresponding odd number itself. Recall that Nicole described the odd integers simply as "straight lines", which was lacking a multiplicative nature. Ben also did not make a generalization about the dimensions of odd numbers. Ben and Nicole were alike in that they were successful in providing a multiplicative type RUC for special cases, though. Ron excelled in that he emphasized the multiplicative RUC for subunits standing for both odd and even numbers. In fact, when I asked him what is common about even numbers, he said that they are all split in two columns. His usage "2 by one half of the number" calls for a relational notation of ordered pair (2,n/2) of linear units where n stands for any even number of the sequence.

Additive Representational Unit Coordination (ARUC)

I observed more than one additive type RUC's which can be described using a functional notation $\sum f(i)=g(n)$ where areal quantities f(i)'s are being summed from 1 to n (number of addends) and i is the stage number (ordering number for the addends).

1) Equal Addends

   These are the addends describing a composite number rectangle. With the functional notation, $f(i)=c$, for all i. All PST produced this type.

2) Irreducible Addends (Type I)

   PST used these addends mostly when dealing with prime number rectangles. This is a special case for equal addends, $f(i)=c$, for all i, with $c=1$.

3) Symmetric Addends

   This type came from Ben's work on the summation of odd integers activity. Ben used these addends to describe the odd integers as symmetric L-shapes (Figure 2). For each symmetric L-shape, there are three addends only, i.e., $n=3$. One of these addends is equal to 1, and the remaining two addends are equal to each other. In other words, with the functional notation, one can write, $f(2)=1$, $f(1)=f(3)$. For example, for the case of areal 9, which denotes an odd integer, $f(2)=1$, $f(1)=f(3)=4$. Note that $f(1)+f(2)+f(3)=4+1+4=9$, i.e., the odd integer itself.

Figure 2. Growing Square Sequence Based on L-Shape Subunits.

4) N+(N-1) type Addends

These are the addends describing a symmetric L-shape odd integer. In this case, n=2. With the functional notation, f(1)=N, and f(2)=N-1, i.e., the addends differ only by 1. John, Stacy, and Ben explained their ideas using this type when working on the summation of odd integers activity.

5) (N+1)+(N-1) type Addends

These are the addends describing a nonsymmetric L-shape even integer. Once again n=2. Only John referred to this type when working on the summation of even integers activity. A nonsymmetric L-shape even integer can be described using the functional notation f(1)=N+1, f(2)=N-1.

6) Recursive Addends

f(i+1) is being added to the previous summation (Nicole). With the functional notation, this can be written as g(n+1)=g(n)+f(i+1).

7) Summed Addends

The addends of the growing rectangle are areal units with different shapes made of magnetic color cubes representing the "area as a sum" part of the summation formula. For example, f(i)=i, f(i)=2i-1, f(i)=i+(i-1), f(i)=2i, for all i, for the addends corresponding to summation of counting numbers, odd numbers, odd numbers, and even numbers, respectively. 3 out of 5 PST (Nicole, Stacy, John) came up with this usage. Ron and Ben, on the other hand, did not care about the color shapes generating the growing rectangle. Instead, they used "Equal Addends" type RUC in expressing the area of the growing rectangle as a sum: Namely they treated the growing rectangle as a composite number rectangle.

8) Random Addends

n can be anything (Stacy). There are many different ways of writing the sum. With the functional notation, f(i)=anything, for all i. And f(i) is not necessarily equal to f(j) for any i≠j, where i, j denote the ordering number for the addends (areal units).

9) Irreducible Addends (Type II)

The area of the polynomial rectangle is written as the sum of irreducible areal units. 4 PST came up with this usage. For instance, for the 2x+y by x+2y+1 rectangle, the irreducible addends are [x^2, x^2, yx, xy, xy, y^2, xy, xy, y^2, x, x, y].

10) "Boxes of the Same Color" type Addends

The area of the polynomial rectangle is written as the sum of the boxes of the same color. As an example, only 1 PST used the areal units [$2x^2$, 4xy, 2x, yx, $2y^2$, y] to generate the 2x+y by x+2y+1 polynomial rectangle.

Mapping Structures (Ordinary and 2-Fold)

Stacy started by making long bars for the even numbers 2, 4, 6, 8, 10. She then added them to generate a growing rectangle sequence based on L-Shape subunits (Figure 3) similar to Ben, Nicole, and John.

Figure 3. Stacy's Growing Rectangle Sequence Based on L-Shapes.

She said that this pattern is similar to the odd integers. The protocol below illustrates this point.

Protocol 3: Stacy's Visual Proof Relating Two Summation Formulas.

S: The only difference is that we have an extra row [She splits the extra row as in Figure 4]

I: So you discovered the formula I guess...

S: Yeah... It would be n squared plus whatever that is [pointing to the extra row she just split]... n... n squared plus n yeah! [very excited]

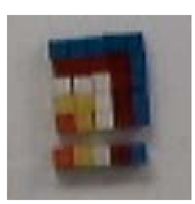

Figure 4. Stacy's Decomposition of the Rectangle into a Square and a Bar.

As can be warranted by the protocol above, Stacy introduces *n* right after pointing to the extra row she just split. She therefore first visually locates both the growing square and the bar, and later on connects these objects to their dimensionalistic properties. Stacy's "extra row" formulation serves as a bridge between the two summation formulas. She just made the figure above and generalized it. She saw the square as an *n* by *n* square, and the bar as *n* (Resulting Addends). Her algebraic generalization was based on a representation (Figures 3 & 4), which was a special case for $n=5$. In Thompson's (1988) words, "To reason quantitatively is to reason about quantities, their magnitudes, and their relationships with other quantities" (page 164). Stacy was reasoning quantitatively by not only relating each L-shape even number subunit to the corresponding odd number component, but by connecting the two separate sequence of growing figures as well.

Stacy's discovery of n^2+n based on letters arising from the special case of $n=5$ made me excited because of her quick generalization in the conjectural process. I wanted to learn more from her about the meanings she would project onto these quantities, as illustrated in the protocol below.

Protocol 4: The Meanings Projected onto the *Resulting Addends* [n^2,n] and Mapping Structures.

I: Okay... n squared plus n... tell me more about that... What units have n squared and n?

S: Well... n square is n times n... so inch times inch it would be inches squared.

I: How about the extra n... is it an area or a length?

S: I don't know...

I: Is it in the area... that n?

S: Yeah... so it would be inches squared... by itself...

I: Does it make sense?

S: Yeah... well added together that has to equal an area so... since you are adding them together they have to have the same units... so it would be inches squared.

I: Okay... Where is the inches squared in n? In that n? [meaning the extra row]

S: It has to be in inches squared...

I: Okay... How do you say that? How do you figure?

S: It's just that it's n times 1... n inch and 1 inch... and then when you multiply them it'd be inches squared.

Stacy establishes arealness for her resulting addend n^2 by multiplying the corresponding same-valued linear quantities. These linear quantities are *n* inches and *n* inches. She demonstrates how the value-wise multiplication of *n* and *n* yields "value" n^2 and the "unit-wise" multiplication of inches and inches yields the measurement unit inches squared. When working on the first activity on prime and composite rectangles, Nicole also did the same thing. She multiplied the values of linear quantities as well as the measurement units attached to those quantities to produce a quantity of a totally different nature (Schwartz, 1988; Thompson, 1988). I infer a *2-fold Mapping Structures* for Stacy's representationally coordinated quantities. Both the ordered pair of values (n,n) and the measurement units (inches, inches) are mapped onto the corresponding value n^2 and the measurement unit in^2 through the multiplication operation behaving as a mapping. A *2-fold Mapping Structure* is slightly different from the *ordinary Mapping Structure* in that the fact that the multiplication mapping operates on both values and measurement units separately as in Stacy's usage "n squared is n times n... so inch times inch it would be inches squared". The ordinary Mapping Structures can be witnessed in Nicole and John's statements ("three inches times seven inches would give me 21 inches squared", "Length of 5 and width of 1 in which case the area would be 5") in their work with the prime and composite rectangles. Since the relational notation of ordered pairs already possesses the measurement units, both types of Mapping Structures have equivalent relational notations, such as (n,n) for the example above. A functional notation which describes both types of Mapping Structures can be written as $f(n,n)\rightarrow n^2$ or with the equality $f(n,n)=n^2$, where, f stands for the multiplication operation behaving as a mapping.

Stacy goes through a hesitation for a very short time in her sense making of the "extra row". Does it stand for a linear or an areal quantity? By her statement "since you are adding them together they have to have the same units... so it would be inches squared", I hypothesize that she establishes the arealness of this aforementioned quantity by deductive reasoning. The steps Stacy follows in her deductive reasoning can be outlined as follows:

1- The big n^2+n rectangle is an areal quantity.
2- The *resulting addend* n^2 is an areal quantity.
3- Therefore, the "other" *resulting addend n* must be an areal quantity, as well.

And finally, once establishing the arealness of the "extra row" quantity via deductive reasoning, Stacy then validates her judgment via inductive reasoning with reference to a *2-fold Mapping Structures*: She maps both the values and measurement units associated with the linear quantities into their areal counterparts as can be warranted by her statement "It's just that it's n times 1... n inch and 1 inch... and then when you multiply them it'd be inches squared." The 2-

foldness of the Mapping Structures arises in that both the ordered pair of values (n,1) and the ordered pair of measurement units (inches, inches) are mapped onto the corresponding value *n* and measurement unit in^2 through the multiplication operation behaving as a mapping. A functional notation which describes Stacy's verification can be written as $f(n,1)\rightarrow n$ or with the equality $f(n,1)=n$, where, f once again stands for the multiplication operation behaving as a mapping.

Notice that Stacy's behaviors in Protocol 4 above are always about the resulting addends generating the growing rectangle which can be modeled via a relational notation of ordered pairs $[n^2,n]$ of areal subunits. To be more specific, Stacy establishes the formation [(n,n),(n,1)]. The n^2, though was one of those big units (growing rectangles) in the context of the previous task on the summation of odd integers, behaves as a subunit in the context of the summation of even integers activity for Stacy. In fact, it is all about how Stacy wants to define this areal quantity by reference to her previous experience with the summation of odd integers activity. Remark that what Stacy establishes is the formation $[n^2,n]$ – equivalently, the formation [(n,n),(n,1)] – and not the formation (n,n+1) which denote equivalent quantities. In fact, when she was working with the "area of the growing rectangle as a product" column on the activity sheet, after providing the answers 4×5, 5×5, 6×7, 7×8, 8×9; I asked her whether these look like the expression n^2+n she discovered above. Stacy established the equivalence of these two formations as illustrated in the protocol below.

Protocol 5: Equivalence of $[n^2,n]$ and (n,n+1) Formations.

I: Does this look like n^2+n? [pointing to Stacy's written expressions on the "area of the growing rectangle as a product" column]

S: No. Does it? I don't think so...

I: Okay... Now I am gonna ask you to factor it...

S: Oh... n times n+1! [She writes $n^2+n=n(n+1)$ and very excited] It works...

I: Does it make sense?

S: Hm hm...

I: What units would you attach to n and n+1?

S: The n and the n+1 are both in inches.

I: What do you think about these as teaching tools?

S: It's cool though... I think it would work with summations. I did not learn the summations in high school though... I guess if you are teaching summations it should work...

Stacy's statements from Protocol 4 and Protocol 5 necessitate the existence of a theoretical construct which I name *Equivalence of Mapping Structures*. There must be an agreement of the ordered pair (n,n+1) of linear units and the ordered pair $[n^2,n]$ of areal units. These two formations can be reconciled via the equivalence of mapping structures. The multiplication operation, which behaves like a function, like a mapping, can be represented using a functional notation such as $f:(n,n+1)\rightarrow n^2+n$. Here, f denotes the multiplication operation which maps the linear units, n and n+1, which can be thought of as combinations of irreducible linear quantities (unit edges), onto the corresponding areal unit, namely n^2+n, which is also the same as the area of the growing rectangle itself. In other words, f acts on the ordered pair (n,n+1) of linear units and maps it onto the areal unit n^2+n. This operation can also be written as $f(n,n+1)=n^2+n$ with an "equals" sign.

Similarly, the addition operation behaves like a function, like a mapping, acting on irreducible areal quantities (unit blocks) or combinations of those. For instance, the function g, which represents the addition operation, acts on the ordered pair $[n^2,n]$ of areal units and maps it into the areal unit n^2+n. Using a functional notation, once again, this can be written as $g:[n^2,n]\rightarrow n^2+n$, or, with the equality $g[n^2,n]=n^2+n$. In other words, though they act on different types of representational quantities, the mappings f and g agree on one thing: That one thing is nothing but the fact that their image coincides (Figure 5). This is the essence of what is meant by "identity" in this research project. Area as a product coincides with the area as a sum eventually, thanks to these mapping structures.

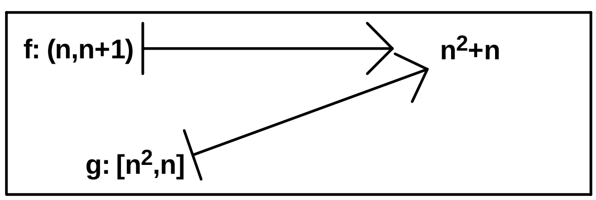

Figure 5, Equivalence of Mapping Structures.

John also made reference to *Mapping Structures* in his work with the summation of even integers. After generating a similar sequence of growing rectangles as in figure 6, he compared same-valued linear and areal quantities, as illustrated in the following protocol.

Protocol 6: John's Comparison of Linear and Areal Quantities and Mapping Structures.

I: How about the 6 and the 4 in the green... [L-shape representing the even number 10] are they areas or lengths?

J: Areas.

I: But this 6 is also the length of this rectangle [pointing to the 6 by 7 rectangle]. Are they [meaning the two 6] the same or different?

J: Different.

I: How are they different?

J: These 6 cubes by itself represent an area [pointing to the vertical part of the green L-shape figure] So this is... It's not just 6... It's 6 and 1.

Figure 6. John's Growing Rectangle Sequence Based on L-Shapes.

John's usage "It's not just 6... It's 6 and 1" can be explained using the *Mapping Structures* analysis model. The 2-foldness in these structures is missing as John does not mention unit-wise mapping (Compare with Stacy's behaviors indicating the 2-fold characteristic of the *Mapping Structures* in Protocol 4 above). John only maps the values of the linear quantities onto the value of an areal quantity. Multiplicative RUC arises from his usage "It's 6 and 1" however that's not the whole story. Multiplicative RUC is only a prerequisite for the construction of a *Mapping Structure*. In fact, John builds on the Multiplicative RUC by his statement "It's not just 6" which indicates the value of the areal quantity under consideration comes alive thanks to the multiplication operation which behaves as a mapping acting on the ordered pair (6,1) of linear units. For a *Mapping Structure* of multiplicative type to exist, therefore, one needs to establish the following conditions:

1- A pair ordering of the values of the linear quantities is mentioned.

2- The multiplication operation behaving as a mapping is acting on the ordered pair of these linear quantities.

3- The value of the areal quantity resulting from the mapping is indicated.

For a *2-fold Mapping Structure* to exist, on the other hand, the conditions above must hold as well as the following:

1'- A pair ordering of the measurement units of the linear quantities is mentioned.

2'- The multiplication operation behaving as a mapping is acting on the ordered pair of these linear measurement units.

3'- The measurement unit of the areal quantity resulting from the mapping is indicated.

In all the activities on magnetic color cubes, Nicole and John are the only students to make use of (ordinary) *Mapping Structures* as opposed to Stacy who excelled in the field with her reference to 2-fold Mapping Structures. Conditions necessitating the existence of ordinary and 2-fold *Mapping Structures* of additive type can be established in a similar manner as in 1, 2, 3, and 1', 2', 3' above.

4- An *n*-tuple ordering of the values of the areal quantities is mentioned.

5- The addition operation behaving as a mapping is acting on the ordered *n*-tuple of these areal quantities.

6- The value of the areal quantity resulting from the mapping is indicated.

4'- An *n*-tuple ordering of the measurement units of the areal quantities is mentioned.

5'- The addition operation behaving as a mapping is acting on the ordered *n*-tuple of these areal quantities.

6'- The measurement unit of the areal quantity resulting from the mapping is indicated.

The identification and the coordination of representational units of different types (multiplicative, additive) associated with color cubes and tiles are important aspects of quantitative reasoning and need to be emphasized during the teaching and learning process. Moreover, mapping structures serve as a bridge linking "area as a sum" and "area as a product" concepts in helping students and teachers make sense of identities on integers and polynomials: Why does the LHS have to be equal to the RHS?

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The Mathematical Preparation of Secondary School Teachers

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Abstract

In the summer of 2007, a group of doctoral students at the University of Georgia gathered to discuss the mathematical preparation of secondary teachers. The group used Mathematics for High School Teachers: An Advanced Perspective by Usiskin, Peressini, Marchisotto, and Stanley (2003) as the catalyst for the discussion. Participants agreed that future teachers need opportunities to examine high school and college mathematics differently from the way they had as students, with specific emphasis on connections, representations, and history. Features of this text that were highlighted in the discussions were the attention topics with commonly held misconceptions, the historical rationales and development of mathematical topics, and the role of mathematical definitions. Group members felt that, depending on one's purpose for using the text and the backgrounds of the prospective teachers, this text could be used, in conjunction with supplemental materials, in a variety of capacities: for a capstone course, a connections course, or a set of replacement mathematics courses.

The Mathematical Preparation of Secondary School Teachers[1]

Introduction

At the meeting of the Association of Mathematics Teacher Educators in January 2007, a group of professors from across the nation gave a presentation on capstone courses. A number of issues brought up in the session were of interest to some of the University of Georgia doctoral students in attendance: the idea of a capstone course in mathematics education, the popularity of a particular text, and the content chosen by different institutions. Most of us were unfamiliar with the concept a of capstone course, having received our undergraduate education from universities with full mathematics education programs. But of even greater interest was the popularity of a book with which we were unfamiliar, *Mathematics for High School Teachers: An Advanced Perspective* by Zalman Usiskin, Anthony Peressini, Elena Marchisotto, and Dick Stanley (2003). Many of the presenters reported using this text for the majority of their class content.

As those present at the session discussed capstone courses with UGA professors and other doctoral students, the actual mathematics preparation of secondary teachers emerged as a primary point of interest. A growing group of doctoral students, who were actively involved in research about mathematical knowledge for teachers at the secondary level, decided to formalize our conversations about the role of capstone courses and the mathematics deemed essential for high school teachers. We developed a seminar to discuss the mathematical preparation of teachers, using the Usiskin et al. text as a guide for the discussion. This summer seminar was open to all doctoral students at UGA and was sponsored by Jeremy Kilpatrick, who had previously used the text in a course at UGA.

The Mathematical Preparation of Teachers

The Conference Board of the Mathematical Sciences (CBMS) released *The Mathematical Education of Teachers* (MET) in 2001. A major point of emphasis was that the preparation needed for teaching mathematics is different from the preparation needed to continue general graduate study in mathematics. This idea differs slightly from the historically held recommendation that those wishing to teach mathematics should earn the equivalent to a bachelor's degree in mathematics. Rethinking teacher preparation was necessary due to the

[1] The author wishes to thank Bob Allen, Kathy Daymude, Sarah Donaldson, Ryan Fox, Gloria Jones, and Margaret Morgan for their thoughts, ideas, participation in the seminar, and ongoing conversations on this topic. I also wish to thank Jeremy Kilpatrick for his guidance and for sharing his personal experiences.

changing face of school mathematics and the expectations placed on teachers. Teachers need to be able to make informed decisions about curricula and instructional strategies that are qualitatively different from what they themselves experienced in school. To meet these needs, teachers should develop:

- Deep understanding of the fundamental mathematical ideas in grades 9–12 curricula and strong technical skills for application of those ideas.
- Knowledge of the mathematical understandings and skills that students acquire in their elementary and middle school experiences, and how they affect learning in high school.
- Knowledge of the mathematics that students are likely to encounter when they leave high school for collegiate study, vocational training or employment.
- Mathematical maturity and attitudes that will enable and encourage continued growth of knowledge in the subject and its teaching. (p. 122)

While some of these recommendations could be accomplished by redesigning present mathematics courses geared toward teachers, the CBMS (2001) advocated a year-long, 6-credit hour capstone course to connect upper-level mathematics courses to the mathematics studied in high school. The course would provide opportunities for future teachers "to look deeply at fundamental ideas of mathematics, to connect topics which students often see as unrelated, and to develop the important habits of mind" (p. 143). Additionally, the course should give prospective teachers "broad historical and cultural perspectives, insight into mathematics learning, and application of technology" (p. 123).

Requiring a capstone course for those intending to teach secondary mathematics is only one of the pathways for connecting undergraduate and high school mathematics proposed by Ferrini-Mundy and Findell (2001). They classify capstone courses under the heading of the "mathematical approach" to connecting content. Also included in this category are shadow courses that are taught alongside undergraduate mathematics courses. The authors caution, however, that the focus on mathematics in this approach may include insufficient pedagogical content knowledge for future teachers. Two other approaches they explicate are an integrative approach and an emergent approach. Integrative approaches weave together content and pedagogy, and emergent approaches begin with the practice, drawing on the mathematics that emerges from real situations. While these two approaches, especially the latter, are rare in the mathematics education programs of which I am familiar, the mathematical approach appears to be quite prevalent.

Mathematical Preparation at UGA

University of Georgia students intending to teach high school mathematics can major in mathematics education or pursue a dual degree in mathematics and mathematics education. Those earning the mathematics education degree alone are required to complete at least two calculus courses as part of their core program and nine upper level content courses, including introduction to higher mathematics, linear algebra, foundations of geometry, modern algebra, statistics, and instructional technology. Two of the three major electives must also be mathematics courses. Students complete education courses on curriculum and teaching methods and may opt to take courses in problem solving, historical and cultural foundations of mathematics, contemporary school topics (including discrete mathematics and modeling), and mathematics in context. Additionally, shadow seminars to accompany two courses, sequences and series (an elective) and modern algebra, are offered to help students see the connections between the college content and high school mathematics. In line with the CBMS recommendations, some of the mathematics courses, including statistics for teachers and the geometry course, have been revised or designed with the needs of future teachers in mind.

Doctoral Student Seminar[2]

Armed with our own personal experiences of being students and mathematics teachers, we began the seminar by reading selections from Ferrini-Mundy and Findell (2001) and Usiskin (2001). Participants were also encouraged to read selections from the MET report (2001). The goal for the readings was to acquaint seminar participants with a historical background for the discussions, with particular attention to rationale for the need for the seminar and understanding of Usiskin et al.'s purposes in writing their text.

As mathematics teachers, and as mathematics teacher educators, each participant brought with him unique perspectives and ideas as to what mathematical preparation was necessary for secondary school teachers. A commonly voiced belief was that teachers needed to complete the traditional mathematics courses, or at least two years beyond what would be taught in high school, followed later by pedagogy. Others, referring to Ma's (1999) work, questioned the needed

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Although this is a report of a group participation seminar, the author accepts responsibility for the ideas expressed in this paper. In many instances, disagreements were voiced. While it is the author's purpose to present an accurate view of the discussions, it is possible that participants' views were misunderstood. The author apologizes if any participant views are inaccurately presented.

for higher mathematics, advocating, rather, instruction of the deeper understandings of high school mathematics. Regardless of the overall beliefs expressed, participants agreed that future teachers need opportunities to examine high school and college mathematics differently from the way they had as students, paying particular attention to connections, representations, and historical development. As the introductory session drew to a close and we began discussing the content presented by Usiskin et al., a series of questions—which remained in our minds, but were never fully discussed—arose: Could undergraduate courses, designed solely for teachers, emphasizing both content and pedagogy, be developed to replace current undergraduate mathematics requirements? What would such classes contain? Could the Usiskin et al. text be used in such courses? In reflecting, I pose an additional question: What impact would a change in preparation such as this have on the view of teaching as a profession?

Background on Mathematics for High School Teachers: An Advanced Perspective

Usiskin (2001) outlines his conception of "teachers' mathematics" as comprising three broad areas: concept analysis, problem analysis, and connections and generalizations. More specifically, these three areas include:

(a) ways of explaining and representing ideas new to students, (b) alternate definitions and their consequences, (c) why concepts arose and how they have changed over time, (d) the wide range of applications of the mathematical ideas being taught, (e) alternate ways of approaching problems with and without calculator and computer technology, (f) extensions and generalizations of problems and proofs, (g) how ideas studied in school relate to ideas students may encounter in later mathematics study, and (h) responses to questions that learners have about what they are learning. (p. 3)

To address the needs of teachers' mathematics, the goals of the text were to provide more material than could be taught in one or two courses for upper level undergraduates or graduate students that focuses on mathematics, not methods, relevant to the classroom. One of the text authors' hopes was to move "towards a set of canonical courses in the field" (p. 4) that provides content and perspectives that are important for secondary teachers.

Although seminar participants tried to keep the authors' intentions and goals for *Mathematics for High School Teachers: An Advanced Perspective* in mind during our discussions, we also tried to see how important we deemed specific topics for secondary teachers. The text authors do provide suggestions for specific chapters to be taught, given different foci for possible courses that would use the text, but we were attempting to analyze the entire text as if we had unlimited time with our potential students. An obstacle we repeatedly faced, just as we face when planning any curriculum or reviewing any materials, was our conception of the mythical student who would be enrolled in a course that uses the text. Should we consider those who had completed the required mathematics courses or should we attempt to replace the required courses with this text? Which required courses should we assume to have been completed? Are all courses equal? That is, if a student completes modern algebra at one university, will the same content have been taught at a different university? My personal decision was to consider prospective teachers attending the University of Georgia, or any other university, who had completed up through college geometry and modern algebra. Further, my guiding question as I reviewed the text was, "What knowledge, or treatment of a topic, would be beneficial for prospective secondary teachers?"

Reviewing the Text

As we read and worked through bits of each chapter, we generally decided that, for each chapter used, the entire chapter should be read by the prospective teachers; however, some content was definitely more essential than other. We attempted to point to topics we felt were treated well, areas we felt were lacking, and, if possible, identify sources that could be used as supplemental to this text. We also suggested exercises for discussion, class presentations, or homework; however, we do believe that such decisions should be based on one's particular students[3]. The remainder of this section highlights the discussions that took place. Note that, although almost the entire text was addressed in the seminar, only aspects that sparked conversation or disagreements are detailed here.

Chapter 1: What is meant by "an advanced perspective"? As the goal of this chapter was to introduce prospective teachers to the ideas of concept analysis, problem analysis, and mathematical connections, the seminar participants felt that this chapter was essential to helping students understand the perspective to be taken in this text. Not only did we see the value in the approach taken in the chapter, but we were also impressed with the content and the chapter problems. We believed that secondary teachers could benefit from discussing or investigating the majority of the chapter problems.

Chapter 2: Real numbers and complex numbers. Seminar participants believed that prospective teachers should understand how the number systems are built up; however, it may not be essential for teachers to be able to prove that development. The historical connections, especially if students do not take other history courses, were especially helpful for both

[3] A list of suggested problems is available from the author by email.

prospective teachers and their future students. We specifically saw value in chapter problems that required teachers to dig deeply into high school mathematics. An activity we found particularly thought-provoking was developing an accurate Venn diagram of the partitions of the real numbers (cf. Usiskin et al., 2003, p. 43).

Chapter 3: Functions. Teachers should understand how functions permeate school mathematics. The attention to representations, connections, and technology, as well as to conceptual understanding of what it means to be a function, was felt to be important. This chapter included a multitude of problems that could also be used with high school students, particularly exercises on discontinuities, common functions (including piece-wise and general step functions), and composition and inverses of functions. We did feel that the topics of fitting linear, exponential, and polynomial functions to data could be better addressed with other methods. Incorporating the use of the TI-83, Fathom, or Excel would better connect the mathematics to high school mathematics. An additional resource with an in-depth treatment of fitting polynomials to data is *Mathematical Connections* by Al Cuoco (2005).

Chapter 4: Equations. We felt that the majority of this chapter could serve as a good refresher of high school mathematics topics as well as address misconceptions about equations that high school students may hold. Of interest was the authors' discussion of the equality, the treatment of the equal sign in school mathematics, and notational issues for equality versus geometric congruence.

Chapter 5: Integers and polynomials. Much of the content found in this chapter may be found in an introductory number theory course or perhaps in an abstract algebra course, particularly if the Shifrin (1996) text is used. One point of division within our group was how we would treat Unit 5.1: Natural numbers, induction, and recursion. Some participants suggested reviewing the first three sections of the unit on the basics of recursion and induction and skip the "extended analysis" of an induction situation, determining the number of regions created by *n* non-concurrent and non-parallel lines. Others felt that mathematics majors or education majors who had completed a few high level mathematics courses would have sufficient knowledge and understanding of induction and would, therefore, garner greater benefits from the problem solving required of the task.

The remainder of the chapter is highly proof-based. Because students may have completed mathematics courses that address divisibility of integers and polynomials, instructors must assess their students' needs in determining what is important to address. Those of us who completed

Shifrin's (1996) abstract algebra found little from units 5.2 and 5.3 that we deemed essential to teach in an additional course. The one exception is section 5.2.5, the base representation of positive integers. This algebraic approach to place value could aid students in deepening their understanding of place value and of the decimal number system, in particular. An enhanced discussion could include historical approaches to place value, counting methods in different cultures, and the use of different bases in technology (cf. Eves, 1990).

Chapter 6: Number system structures. As with chapter 5, much of this content appears to reinforce abstract algebra topics, with few additional connections to the actual mathematics found in high schools. However, the applications of modular arithmetic to cryptology and calendars may not have been seen by teachers in traditional mathematics courses. As these are topics that may engage their future students, exploring them may be beneficial.

Chapter 7: Congruence. Three major ideas in chapter 7 proved fertile ground for discussion. One would hope that teachers who complete a college geometry course would have experience with and understand the implications of Euclid's *Elements*, specifically the 5 postulates. However, teachers may have not fully explored the affordances of different definitions and assumptions. Our discussions repeatedly called into question our own definitions for geometric figures, such as cylinder, trapezoid, and rhombus. In examining high school texts, we found that they, too, sometimes differed in their definitions and in what they classified as theorems and postulates. Investigating different texts' definitions of common terms could lead to a discussion of the benefits and strengths of different characterizations.

Another point of interest was the transformational approach to congruence presented by the text. High school texts also differed in their approaches. Some discussed congruence in terms of transformations whereas others conceptualized it in terms of equal measures. Teachers must be aware of how texts treat high school concepts and know where to look for materials that may provide a different approach. This type of awareness, or curricular knowledge (Shulman, 1986), is not likely inherent in prospective teachers; it must be developed.

A final note on this chapter, which also applies to others, is the role of geometry programs such as GSP. Seminar participants differed in their beliefs about the degree to which these programs should be used. All agreed that dynamic software programs are valuable for investigations and testing conjectures; disagreement arose in determining if theorems could be proved with the software and in how much reliance should be placed on such programs. It was suggested that prospective teachers also need experience with a compass and straightedge, and they should know how the software constructions work. Further, prospective teachers should have opportunities to explore a variety of dynamic software programs.

Chapters 8–11. The discussion of this text occurred during summer semester, and as is the case with many courses, we found ourselves with too little time to adequately accomplish our full agenda. Therefore, the discussion on chapters 8–11 was limited but reinforced our previous assessments of the text in terms of a deeper awareness of importance of definitions, providing links to high school mathematics that prospective teachers may have not visited since they were high school students, and situating mathematical topics in historical contexts.

Next Steps

Seminar participants expressed interest in continuing the discussion of *Mathematics for High School Teachers: An Advanced Perspective* in future semesters. In addition to engaging in deeper conversations about the last chapters in the text, we would like to design courses with specific emphases, supposing specific prerequisite knowledge, for prospective teachers. One potential design could be that of a capstone course to serve the needs of mathematics majors who seek certification. Another design could characterize a “connections” course, with the goals of connecting college mathematics to high school mathematics and refreshing prospective teachers’ knowledge of high school mathematics content. A third design could include a number of courses, namely a set of courses designed to replace present mathematics requirements with courses that not only address the high level of mathematics taught in the traditional courses but also the pedagogical connections needed by teachers. All three of these design paradigms seem to be aligned with the assertion that teachers need to understand qualitatively different mathematics from that needed by mathematics majors.

Closing Thoughts

High school mathematics teachers need a deep understanding of mathematics. Most teacher educators would likely agree with this statement. *Mathematics for High School Teachers: An Advanced Perspective* by Usiskin et. al (2003) is one text that may prove helpful in deepening prospective teachers’ mathematical knowledge. The authors hold high expectations for users’ previous mathematical knowledge and ability to engage in high level mathematics. A few of the features highlighted in the doctoral student seminar were the attention topics that may aid teachers in correcting or avoiding the formation of student misconceptions, the historical rationales and development of mathematical topics that help build connections within—and to topics outside—mathematics, and the importance of one’s mathematical definitions to their ability to extend ideas.

No one text can meet all the mathematical preparation needs of high school mathematics teachers. Topics that we deemed important for teachers that received little-to-no attention in this text are logic, truth tables, statistics, and probability. Additionally, we believe that teachers need greater knowledge of historical and cultural topics in mathematics than what is provided in the text; a history course would benefit teachers as well as their future students. Technology should be used, where possible, in teachers' preparations; teachers should be aware of uses, advantages, and limitations of various instructional technology. Integrating this type of knowledge with their deepening understanding of mathematics may provide prospective teachers with ideas of how to use technology to engage their students and facilitate student mathematical understanding. Finally, we felt that mathematics teacher educators could include pedagogical ideas, not a focus of the text, into classroom discussions, developing more of an integrative approach to mathematical preparation, rather than a purely mathematical approach as is the focus of the text.

Just as teachers need curricular knowledge, including an awareness of how to find information or alternative treatments of mathematics topics, teacher educators should also develop an arsenal of instructional materials. Decisions about the specific content to be addressed in a given course with a particular group of students should be based on the goals of each course, the instructor's specific goals, and the needs of the students. However, the UGA doctoral students who participated in this seminar on the mathematical preparation of high school teachers feel that the Usiskin et. al (2003) text could be a valuable addition to the libraries of mathematics teacher educators. The text, and others written with similar intentions, can serve as catalysts for conversations about the mathematical preparation of high school teachers: how it should be accomplished and what should be taught.

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Using Technology to Design Teaching Modules in Mathematics and Science

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Abstract

Technology is changing the way in which mathematics and science are taught, and this radical transformation in teaching is causing teachers to take a closer look at how lessons are designed. In an effort to demonstrate how to design instructional modules using technology, this paper will include the following: 1)A review of the National Educational Technology Standards for teachers to establish a framework for the development of the teaching modules; 2)instructional designs and techniques with special emphasis on multiple intelligence and critical thinking skills; 3) strategies and techniques for infusing technology into a standard based curriculum; and 4) an analysis of the evaluative data completed by pre-service teachers to determine the effectiveness of the modules. Middle and elementary pre-service teachers at a historically Black university are required to take a course, Integrated Methods of Teaching Science and Mathematics and the previously mentioned instructional modules were used in this course. The learning modules provided the pre-service teachers with technology enhanced experiences that were aligned with the national science and mathematics standards.

Pre-service teachers were asked to complete five modules which included both formative and summative assessments. They were also asked to complete a survey to obtain their perceptions of the teaching strategies, their performance, the use of tools and assistance, and comfort with using the computer. The only statistically significant correlation ($p<.01$) was between performance and teaching strategies.

During the exit interview for the course, five of the pre-service teachers indicated that they liked the web-enhanced course and that it was good for their personal schedules; however, all the students had a concern about the required assignments and the amount of time required to complete them. All but one of the students indicated that they would take another web-enhanced course. Teaching modules that incorporate the use of technology are an excellent way to meet the learning needs of a diverse population

Using Technology to Design Teaching Modules in Mathematics and Science

Introduction

Technology is changing the way in which mathematics and science are taught, and this radical transformation in teaching is causing teachers to take a closer look at how lessons are designed. Traditionally, mathematics and science teaching have included lectures, problem solving activities and limited laboratory experiences. One way to expand this traditional format of teaching is to develop modules that allow students to interact with computers, take virtual field trips, communicate via email with teachers and other students, observe and perform simulations of selected procedures, and to complete homework assignments online. In an effort to demonstrate how to shift from a teacher-centered instructional design to a more student-centered one, this paper will include the following: 1) A review of the National Educational Technology Standards for teachers to establish a framework for the development of the teaching modules; 2) instructional designs and techniques with special emphasis on multiple intelligence and critical thinking skills; 3) strategies and techniques for infusing technology into a standard based curriculum, and 4) an analysis of the evaluative data completed by pre-service teachers to determine the effectiveness of the modules.

Rationale

The student population is changing in our Pk-12 schools in that they are no longer sit still and quiet types. They are students who have been interacting with technology and electronics either by playing video games, operating a Digital Versatile Disc player, or setting the controls on high definition televisions prior to entering school (Thoms, 2007). One way to meet the academic needs of these high tech students is to design lessons that incorporate the use of technology. This design will allow teachers to meet the needs of the students while shifting from a teacher-centered instructional design to a more student centered one that supports rigorous and challenging instruction. Mckenna, Avery, and Schuhardt (2000) report that there are many benefits for infusing technology into the teaching of science and mathematics, and some of them are: Expanding the opportunity to meet the needs of students through more individualized learning opportunities, improving assessment through immediate feedback, enhancing communication between students and teachers, providing more opportunities for problem solving activities, and the development of critical thinking skills. Students sometimes have trouble grasping concepts when taught in a large group, and many of them become passive and fail to engage in learning experiences. With the use of teaching modules that incorporate the application of technology students are given the opportunity to work in small groups or individually which lowers the level of frustration and decreases the embarrassment of participation when placed in a larger group. When students complete assignments using technology, they receive immediate feedback and the results of the assessment indicate whether the students are making progress in mastering selected content (Staudt, 2002). This type of feedback is necessary in order for students to correct their mistakes and continue the learning process. It is also a good way to chart student progress. Students must have the opportunity to engage in activities that require them to think critically, and this can be done through inquiry experiences that incorporate the use of technology. There are many inquiry models, but this paper will be limited to two: Bybee's (2002) Five "E" Model and a simple format of the Scientific Method. Mckenna et al. state that we are presently living in the information age and that there is a need to communicate constantly with other individuals in order to obtain information as well as to give information. Technology allows teachers and students to communicate during both the school and after the school hours via email, text messages, teleconferencing, facsimile, and telephone. All of these are ways to improve communication between students, teachers, and the home. These benefits lend themselves to improving the academic performance of students while allowing teachers to shift an educational paradigm from traditional teaching to a more student-focused environment.

Questions

Mckenna et al. (2000) write that there are many benefits of infusing technology into classroom teaching; therefore, this paper will examine the following questions: 1) What is instructional technology? 2) What historical developments influence instructional technology? 3) What are the National Education Technology Standards and how do they impact instructional planning? 4) How do learning theories support the use of technology in the development of learning experiences? 5) How to develop instructional modules that utilize technology in science and mathematics? 6) What impact did the modules have on the performance of the pre-service teachers?

Definition of Instructional Technology

The Association for Educational Communication and Technology (AECT) (1994) reports that instructional technology is the theory and practice of design, development, utilization, management and evaluation of processes and resources for learning. The development of learning activities using technology should be theoretically sound and should employ the principles of the theories in the implementation of learning experiences. Hug (1978) writes that instructional technology is the identification, generation, application and evaluation of processes which create a purposeful structure for developing learning experiences from available resources. Lever-Duffy, McDonald and Mizell (2003) define instructional technology as any media that is used in teaching and it might include printed media, models, projected and non-projected visuals, audio, digital and video media which includes computers, computer peripherals, and computer software. For this paper, instructional technology is inclusive of the media, materials, teaching strategies, instructional designs and theories that support learning using technology as a tool.

History of Educational Technology

Instructional technology began to influence the teaching and learning process during the 1600s with the invention of the quill pens and slates that were used to teach students how to write. This material impacted the way in which teachers taught students how to cipher letters (Lever-Duffy et al, 2003). Technology became more sophisticated and paradigms shifted in the early 1900s with the invention of the movie projector and later the filmstrip projector. These inventions made classroom instruction better for the visual and auditory learners. In the mid 1900s the television was invented and was later used for instructional purposes. Programs were developed by the Public Broadcasting Station that introduced a form of technology that allowed for the design of lessons not only for the visual and auditory learners but also the musical rhythmic and kinesthetic learners. Students were able to move and follow rhythmic patterns pictured on the television screen. This innovation led to the Public Broadcasting Act which established the Public Broadcasting Service and National Educational Radio in the late 1960s. The invention of the personal computer was the beginning of the information age and a new type of instruction, Computer Assisted Instruction (CAI) which was a drill type of instruction that provided immediate feedback to students. Later, the use of digital videos, computer discs, multimedia tools, interactive video, teleconferencing, and the internet expanded instructional technology and made it appealing for all learners regardless of their learning preference (Anglin, 1991). Instructional technology continues to influence the way in which teachers design and implement lessons so that students will have good positive challenging learning experiences that foster success in the classroom.

National Education Technology Standards

The purpose of the National Education Technology Standards Project is to develop goals and objectives that will guide teaching and learning using technology as a tool (2005). The standards delineate both acceptable teacher and student performances that are required for them to function in an information rich society. This paper will focus on teacher performance standards which ultimately will impact student performance. Middle and elementary pre-service teachers at a historically Black university are required to take a course, Integrated Methods of Teaching Science and Mathematics and the previously mentioned instructional modules were used in this course. According to the National Education Technology Standards, teachers must be able to demonstrate the knowledge and skills of technology operations and concepts; plan and design learning environments and experiences; implement curriculum plans that include methods and strategies for applying technology that maximizes learning; apply technology to facilitate a variety of effective assessment and evaluation strategies; use technology to enhance their productivity and professional practices, and understand the social, ethical, legal, and human issues surrounding the use of technology in PK-12 schools and apply these principles in their selected work places. Teachers must know what technological resources are available to them and they must have the knowledge and skills to use them. In the implementation of the modules for this study, pre-service teachers needed to have good knowledge of the use of computers, Microsoft Office Software, calculators, and search engines. Students who were challenged technologically received support from the university's technology and resource center. The use of technology allows teachers to develop learning experiences that meet the needs of diverse student populations. Through the use of modules in the methods course, student needs were met because the modules allowed for various learning styles, allowed students to work at a time that was convenient for their schedules, provided opportunities for both interpersonal and intrapersonal experiences, and enhanced their research skills through the application of selected inquiry models. In addition to completing the five required modules, the pre-service teachers were asked to create developmentally appropriate models for selected grade levels that they could use during their pre-service teaching experience. The learning modules provided the pre-service teachers with technology enhanced experiences that were aligned with the National Science and Mathematics Standards. Pre-service teachers must know the content of their subject areas as well as how to use technology; therefore, they were required to complete assignments that demonstrated that they have adequate knowledge to teach the content in their discipline of choice. Assessment was determined academically by the portfolio that the pre-service teachers completed; however, they were given an opportunity to evaluate the modules and express their motivation for completing them through a teacher made survey. The completion of the modules required the pre-service teachers to communicate with the instructor and other students electronically. This communication tool was very beneficial because it allowed the pre-service teacher access to the instructor daily at unlimited times. This tool also lowered the level of frustration for the pre-service teachers who were experiencing challenges completing the modules.

There are legal and ethical issues that must be considered when using technology. A review of the university's position on these two topics was shared with the pre-service teachers, and a copy of the policy was included as a part of the syllabus. These standards were developed to guide the use of technology as well as to help prepare students for the technological challenges that they will face as classroom teachers.

Learning Theories

There are many theories that impact learning and student performance. The Constructivists believe that learning is based on prior experiences and is influenced by one's culture; the Behaviorists think that learning results from being rewarded in order to avoid punishment, and the Cognitivists think that learning is the result of understanding new ideas that result from critical thinking. Gardner (1993), a Constructivist, proposed the Theory of Multiple Intelligence in 1983 in which he identified nine innate capabilities: Verbal-linguistic, visual-spatial, bodily-kinesthetic, logical-mathematical, rhythmic-musical, intrapersonal, interpersonal, naturalist, and existentialist. He wrote that all students have a combination of these intelligences and that teachers should teach in such a way that would encourage them to develop the preferred abilities. Gardner proposes that teachers use a variety of ways and intelligences in presenting lessons so that the individual learning needs of all students are met. The use of technology is one way to differentiate lessons so that they appeal to a variety of learners and their preferred intelligence.

One way to implement Gardner's theory (1991) is to use the tool of technology to develop instructional modules that require the use of multiple intelligences. Lessons may be designed that incorporate the use of two or more of the intelligences, and students should have an opportunity to select the activities in which they would like to engage based on their preferred intelligence. In addition, interdisciplinary units may be developed in which students may be

given assignments that require the use of all of the intelligences in order to strengthen those areas in which they might feel challenged to perform.

Differentiating Instruction

Not only are teachers required to make lessons appropriate for all student intelligences, they must develop lessons that are rigorous and challenging. This can be done by designing lessons that apply Bloom's Taxonomy for classifying objectives. Bloom proposes a hierarchy for grouping objectives, and the lowest level of the hierarchy is knowledge in which students are asked to recall information; comprehension which requires the interpretation of information; application requires the use of information to solve problems or understand new situations; analysis requires the breaking down of wholes into parts and recognizing parts within the data set, synthesis allows for putting data sets together to generate new ideas, and evaluation requires making judgments (Bloom, 1956). Bloom's hierarchy is used as a guideline for developing the objectives for each of the modules, and this was done to make sure that the objectives were rigorous and challenging for all students.

Developing the Modules

The modules were developed using Microsoft Office and WebCT (Vista) software. Microsoft Word was used to develop the reading passages and other print assignments. Microsoft Power Point was use to develop lectures and some assignments. Microsoft Excel was used for developing data bases and organizing data for analysis. A video camera was also used to tape demonstrations of laboratory experiments which were posted as a part of the modules. All of these requirements were posted on the class WebCt (Vista) page as modules for the pre-service teachers to complete. Other activities were linked to the internet via the use of a uniform resource locator (URL). Assessment and survey instruments were developed using WebCT (Vista) which also graded and organized the results of the data. Microsoft Office Software and WebCT (Vista) were the technological tools that supported the development of the modules.

The pre-service teacher population consisted of eight females and two males who were classified as upper-class students based on the number of credits that they had earned. All of the students attended a required orientation in the computer center and learned how to navigate using WebCT (Vista). Students were also given directions for completing the modules and several resources were identified where they could obtain assistance if needed.

In an effort to help pre-service teachers obtain the content necessary for teaching elementary and middle grade mathematics and science as well as to demonstrate for them how to incorporate the use of technology in their teaching strategies, the modules were developed. Five modules were developed by the teacher with the assistance of a colleague, and these modules were selected based on the requirements for meeting the knowledge standards as explained by the National Science Standards (1-6, and 8), National Educational Technology Standards (1-6), and the National Mathematics Standards (4, 5, 7, and 12). The instructor working with a colleague selected the content for the course and developed the modules using the national standards as a guideline. The modules are: Safety, Measurement, Flowering Plants, Body Parts, and Earth Science. Each of the modules contained objectives that covered the range of Bloom's Hierarchy (1956); a reading selection to explain the content so that students would have knowledge of the concept; a Power Point Presentation, laboratory experiences both virtual and actual; virtual field trips and other internet activities, an art or music activity, and an assessment activity. The modules were interactive, promoted critical thinking and allowed for individual learning style or intelligence preference as delineated by Gardner (1983). Pre-service teachers were to have all of the modules completed by the mid-semester grading period. After completing the modules, pre-service teachers were to complete a survey to assess the effectiveness of the modules as well as to provide feedback on how they could be improved.

The objectives were written in measurable terms and moved from very simple tasks to more complex ones. For example, in the module, Body Parts, a simple objective was: Identify and label all parts of the heart; and a more complex objective stated: Research a prosthesis that is used to help with the flow of blood through the heart and determine the effectiveness of the devise that you researched. These objectives moved from a recall activity (identify) to an application experience (research) to requiring an evaluative response (determine the effectiveness). These examples are used to demonstrate for the pre-service teachers how to develop meaningful and challenging learning experiences for their students.

Activities were organized and followed Gardner's Theory of Multiple Intelligence (1983) and Bybee's Five E Model (2002). Engaging, exploring, explaining, elaborating, and evaluating are the Es' that make up the Bybee Model. The first E is engaging, and pre-service teachers using the modules were engaged by first of all reading the objectives and determining what was required to complete it. In addition to the objectives, each module contained a minimum of one selected reading passage which explained the concepts covered in the module. Students were asked to read the selection and then write a discussion of the passage following guidelines given by the instructor (Verbal Linguistic). The second E is exploring in which students were required to complete a laboratory/hands-on-mind-on activity (Bodily Kinesthetic). For example in the module, Body Parts, one of the requirements for this section was to develop a model of their heart and to scale it to the size of their heart based on their weight, height, and the size of other body parts. They also observed a virtual dissection (Visual Spatial) of the human heart as well as a heart transplant. At the end of this section, they were asked to write a poem about the flow of blood through the heart, set it to a beat, develop a song, and teach it to the other pre-service teachers (Musical Rhythmic). The third E is explaining and these activities allowed pre-service teachers to explain the findings of their investigations to others in both small and large group settings (Interpersonal) and to clarify any misunderstanding about the knowledge required to complete the assignment. This part of the process is critical because the assignments so far have been at the recall and application levels of Bloom's Hierarchy, but the next E, elaborate, will require the pre-service teachers to analyze and evaluate data. The Body Part module asks the pre-service teachers to extend their knowledge of the heart by analyzing the model of the heart that they constructed and the way in which the blood flows through it. Then the module moves to the, "what if question", by asking them to consider the flow of blood if there is blockage in one of the vessels or chambers. The pre-service teacher has the choice of selecting the type of blockage. It is at this point that they are asked to develop a devise and determine if it will help to relieve the blockage and allow the blood to flow smoothly through the heart. This type of activity requires the pre-service teachers to transfer the knowledge to a new situation and to dig for deeper meaning of the concepts. The fifth E is evaluation and evaluations were both formative and summative that were done electronically. In order to complete the module, pre-service teachers had to navigate through the module using WebCT (Vista), use a search engine to do research, use Microsoft Office Software to develop the portfolio, and complete the assessments.

Each module required the development of an electronic portfolio showing the products developed, the assignments completed, the assessment results, and the reflection of the teaching experience. These four pieces were a must for all portfolios; however, if a student wanted to add additional information, he or she could.

Results

The purpose of this paper is to examine how technology could be used to design teaching modules in mathematics and science and to determine the effectiveness of the modules as perceived by the pre-service teachers. The modules were developed using Microsoft Office and WebCT (Vista) software. There were ten pre-service teachers enrolled in the course, all of them African-American. All pre-service teachers completed the five assessments and the mean scores for the electronic portfolio (See Appendix A) for each of the modules is as follows: Safety 83.78 with a standard deviation of 8.41; Measurement 77.38 with a standard deviation of 9.08; Flowering Plants 92.43 with a standard deviation of 6.08; Body Parts 87.74 with a standard deviation of 6.41, and Earth Science 88.60 with a standard deviation of 7.90. All of the means showed moderate variation as indicated by the standard deviation. All of the students made a passing score on the modules with 100 being the highest score and 70 the lowest.

In addition to completing the assessments for the modules, students were asked to complete the OMIM Survey (See Appendix B) to determine their perception of the course and the use of technology. The survey was scored on a Likert (1932) type scale from 1 to 5 with 5 being strong agreement and 1 being strong disagreement. The survey consisted of 15 items and was divided

Table 1 Means and Standard Deviations for Modules

Module	Mean	Standard Deviation
Safety	83.78	8.41
Measurement	77.38	9.08
Flowering Plants	92.43	6.08
Body Parts	87.74	6.41
Earth Science	88.60	7.90

into the following sections: Section 1) comfort with using the computer; section 2) performance; section 3) teaching strategies, and section 4) tools and assistance. Students completed the surveys and submitted them electronically.

The mean for section 1 is 4.40 with a standard deviation of .894; section 2 has a mean of 3.97 with a standard deviation of .753; section 3 has a mean of 4.53 with a standard deviation of .548, and section 4 has a mean of 4.30 with a standard deviation of .610. The sections showed little variance about the mean indicating that the scores were close together.

Table 2. Descriptive Statistics for the Survey Sections

Section	N	Minimum	Maximum	Mean	St. Dev.	Pearson Correlations ($p<.01$)
Comfort With Using the Computer	10	3.00	5.00	4.40	.894	.2520
Performance	10	2.00	5.00	3.97	.612	1.00
Teaching Strategies	10	4.00	5.00	4.53	.115	.8076
Tools and Assistance	10	3.00	5.00	4.30	.346	.4794

Several Pearson Product Moment Correlations were done to determine the strength and direction of any association between performance and comfort with using the computer, teaching strategies and the use of tools and assistance. When pre-service teacher performance was compared with teaching strategies, the value obtained for r was .8076 which showed a strong positive relation between the two variables that was statistically significant ($p<.01$). There was a very small positive correlation, .2520 between performance and comfort with the using the computer which was not statistically significant ($p<.01$); and a small positive correlation, .4794 between performance and tools and assistance which was not statistically significant ($p<.01$). The only statistically significant correlation was between performance and teaching strategies. During the exit interview for the course, five of the pre-service teachers were asked the following questions: What did you like about the course? What changes would you make in the course? Would you take another web-enhanced course? The students' responses are as follows:

Question 1, what did you like about the course?

Student 1: I liked being able to submit assignments on WebCT and being able to work on assignments when I got ready.

Student 2: I liked the chat room discussions and the lecture notes that I could get from WebCT because I could check my notes against the lecture notes to make sure that I had them right.

Student 3: I liked the modules and being able to do them in my room.

Student 4: I liked the flexibility of not having to come to campus and getting my work done at the same time.

Student 5: I liked being able to send my work in without going to class and emailing everybody at one time.

Question 2: What change would you make to the course?

Student 1: I felt like I was rushing to get my work done so I think if I had had more time to do it I would have done better.

Student 2: I didn't have time to do all of the assignments. I think that I would give the students more time to complete assignments.

Student 3: I wouldn't give so many assignments, and I would let the students choose the ones that they wanted to do.

Student 4: For the students completing the modules, they would not have to come to class and for the students who wanted to come to class, they could. It was a lot to have to do all of that work and keep up with other assignments.

Student 5: I would give the students more time to do the work.

Question 3: Would you take another web-enhanced course?

Student 1: No.

Student 2: Yes.

Student 3: Yes.

Student 4: Yes.

Student 5: Yes.

All of the students agreed that they liked the web-enhanced course and that it was good for their personal schedules; however, all the students had a concern about the required assignments and the amount of time required to complete them. All but one of the students indicated that they would take another web-enhanced course.

Conclusion

The purpose of this paper is to examine how technology could be used to design teaching modules in mathematics and science and to determine the effectiveness of the modules as perceived by the pre-service teachers. Modules were designed using a cognitivist approach that focused on the Theory of Multiple Intelligence as developed Gardner (1993). Activities were included in the designs that allowed for a variety of learning and cultural styles and preferred intelligence. The modules were a good way to demonstrate to the pre-service teacher how to incorporate the use of technology into the development of learning experiences. In addition to incorporating the technology, teachers must make sure that the instructional activities are standards based and academically challenging. Again this was demonstrated in the modules in that the National Education, Mathematics, and Science Standards provided the foundation for the development of the modules. Critical thinking strategies were required to complete the inquiry segments of the modules which meant that the pre-service teachers had to apply prior knowledge to complete many of the assignments.

The impact of the modules on the performance of the pre service teachers was determined by the quality of work that they completed and the results of the summative assessment activity. Student performance was very good, and not one of the pre-service teachers received a failing grade. All of them attempted to complete the modules even though, they admitted that they were challenged by the assignments and the amount of time required to complete them. Time allocation for completing the modules will be taken into consideration if the modules are used again.

The design of the modules using technology was good and had a positive impact on the students as indicated by the strong positive correlation when the Pearson Product Moment Correlations were determined. Teaching modules that incorporate the use of technology are an excellent way to meet the learning needs of a diverse population.

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Appendix A

This rubric format was used to assess students' performance for each of the modules and to determine their mean score for all of the activities required for the module.

Name ____ Score _________

Rubrics for Electronic Portfolio

Evaluative Criteria	**Exceptional**	**Acceptable**	**Developing/Incomplete**
Case Studies	Well organized, accurate information, all components evident, well thought out, creative, and innovative (9-10 points)	Organized, accurate information, most components evident, good thinking, creative and innovative (8-6 points)	Poorly organized, some information accurate, some components missing, not well thought out, some creativity (5-2 points)
Inquiry Project	Title inclusive of variables, purpose stated comprehensively, essential materials listed, procedure contains details, written in 3rd person and past tense, graphs and tables electronically developed, data analyzed statistically and conclusions and comparisons made by referencing the data. (9-10 points)	Title stated, purpose limited, some materials listed, procedure written in past tense, 3rd person, with some details missing and not sequential, graphs and tables limited, data analysis limited and conclusions and comparisons made with minimum reference to the data (8-6 points)	No title, no purpose, lacking materials list, procedure missing some details, not written in past tense, not written in 3rd person, many details missing, graphs and tables poorly done, conclusion not stated (5-2 points)
Video Summaries/Lecture Notes	Well organized, accurate information, all components evident, well thought out, creative, and	Organized, accurate information, most components evident, good thinking, creative and innovative (8-	Poorly organized, some information accurate, some components missing, not well thought out, some creativity (5-2 points

	innovative (9-10 points)	6 points)	
Lab Reports	Title inclusive of variables, purpose stated comprehensively, essential materials listed, procedure contains details, written in 3rd person and past tense, graphs and tables electronically developed, and conclusions drawn by referencing the data. (9-10 points)	Title stated, purpose limited, some materials listed, procedure written in past tense, 3rd person, with some details missing and not sequential, graphs and tables limited, conclusions drawn with minimum reference to the data (8-6 points)	No title, no purpose, lacking materials list, procedure missing some details, not written in past tense, not written in 3rd person, many details missing, graphs and tables poorly done, conclusion not stated (5-2 points)
Assessment Instruments	Formative and summative assessments completed at or before deadline with a mean score of 90% or higher (9-10 points)	Formative and summative assessments completed at or before deadline with a mean score of 75% or higher (8-6 points)	Formative and summative assessments completed at or before deadline with a mean score of 60% or higher (5-2 points)
Article Critiques	Accurately explains the author's purpose, hypothesis, and methodology, analyzes the data and gives statistical significance of the data, explains the validity of the conclusions and makes 3 or more recommendations for expanding the study (9-10 points)	Explains the author's purpose, hypothesis, and methodology, analyzes the data, explains the conclusions and makes 2 or more recommendations for expanding the stud y (8-6 points)	Gives a limited explanation of the author's purpose, hypothesis, and methodology, analyzes some of the data, gives limited explanations of the conclusions and makes one recommendation for expanding the stud y (5-2 points)
Field Trips (Virtual)	Explains the purpose in detail, gives allowance for interactions with examples, targeted audience identified with an explanation, knowledge and content obtained	Explains the purpose, allowance for interactions, targeted audience identified, knowledge and content obtained as a result of the experience,	Lacks purpose, minimum allowances for interactions, no targeted audience identified, does not mentioned knowledge and content obtained as a result of the experience, no reflections regarding credibility of the site, and one recommendation for improvement (5-2 points)

	as a result of the experience, reflections regarding credibility of site, and 3 or more recommendations for improvement (9-10 points)	reflections regarding credibility of the site, and 2 or more recommendations made for improvement (8-6 points)	
Reflective Essay	Well organized, accurate information, all components evident, well thought out, creative, and innovative (9-10 points)	Organized, accurate information, most components evident, good thinking, creative and innovative (8-6 points)	Poorly organized, most information accurate, some components missing, not well thought out, some creativity (5-2 points)
Optional: Artifacts	Excellent selection, excellent visuals, demonstrates knowledge of all of the objectives, clearly presented, has a reflection with many details and examples (9-10 points)	Good selection, good visuals, demonstrates knowledge of some of the objectives, presented in somewhat of a clear format, has a reflection with some details and examples (8-6 points)	Poor selection, lacks visuals, does not demonstrates knowledge of the objectives, presented in a poor format, reflection lacks details and examples (5-2 points)

OMIM/10-11-01

Appendix B

Survey for Web-Enhanced Course

Listed below are several statements that will be used to obtain your perception of a Web-enhanced course. Please respond to them by placing an X in the appropriate space.

Statement	**Strongly Agree (5)**	**Agree (4)**	**Neither Agree nor Disagree (3)**	**Disagree (2)**	**Strongly Disagree (1)**
1. I felt comfortable using the computer.					
2. I felt comfortable using WEBCT.					
3. The web-enhanced course helped me excel in this course.					
4. I enjoyed the flexibility of the web-enhanced course.					
5. The web-enhanced course enabled me to successfully complete all assignments.					
6. The web-enhanced course created additional stress for me.					
7. The orientation session prepared me to use the WEBCT software.					
8. The assignments were challenging.					
9. The lecture notes helped my performance.					
10. The Web page was well organized.					
11. The assignments allowed me to work independently.					
12. I have adequate knowledge to do well in a web-enhanced course.					
13. The professor provided assistance when requested.					
14. The email tool was helpful.					
15. I will take another web-enhanced course from this professor.					

Comments____

Assessing Understanding of Multiplication through Words, Pictures, and Numbers

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Abstract

The objective of this session is to engage mathematics teacher educators in a discussion of how to assess an understanding of the concept of multiplication as an operation and its relationships to other operations. The session will begin with a presentation of a previously published study assessing children's understanding of multiplication as grouping and the relationship between multiplication and addition. The assessment asked a series of problems involving words, pictures, and numbers. The results of the study indicate that the types of problems asked were successful in providing evidence of children's understanding of multiplication. The study also found that a group of third grade children had developed a better understanding of multiplication after just one multiplication unit from Investigations in Number, Data, and Space than a comparable group of fourth grade children had developed from an entire year of a traditional emphasis on memorizing multiplication facts. An interactive discussion of potential uses of this study and its assessment format in teacher education will follow the presentation

Assessing Understanding of Multiplication through Words, Pictures, and Numbers

Introduction

Although much research has shown various problems with curricular overemphasis on facts and skills, O'Brien and Casey (1983a, b) specifically demonstrated that many children who have experienced a "back to basics" curriculum "do not know what multiplication is. They have algorithmic skill but no mathematical knowledge of multiplication" (1983a, p. 250). Nonetheless, many parents and teachers continue to consider memorizing basic facts as the hallmark and primary goal of school mathematics, with a particular emphasis on memorizing multiplication facts during third grade. In contrast, the *Principles and Standards* (National Council of Teachers of Mathematics, 2000) argued that "learning mathematics with understanding is essential" and that research shows "the alliance of factual knowledge, procedural proficiency, and conceptual understanding makes all three components usable in powerful ways" (p. 20).

In a previously published article (Smith & Smith, 2006), from which this presentation is derived, we described an assessment of conceptual understanding of multiplication through words, pictures, and numbers used with two different groups of students at one school. These two groups of students were (a) a group of third-graders from a standards-based classroom immediately following their first unit on multiplication from *Investigations in Number, Data, and Space* (Tierney, Berle-Carman, & Akers, 1998; hereafter referred to as *Investigations*), and (b) a group of fourth graders at the same school who had received a traditional third-grade mathematics experience and had been "certified" by their teacher as having memorized all the traditional multiplication facts.

Conceptual Understanding of Multiplication

"Understand meanings of operations and how they relate to one another" is one of the *Principles and Standards'* three major themes for prekindergarten through Grade 12, and many researchers have explored the details of multiplication and how conceptual understandings of it develop (Greer, 1992; Harel & Confrey, 1994; Hiebert & Behr, 1988; Sowder et al., 1998). This research indicates that, although children typically develop additive reasoning quite naturally, multiplication is much more complex than addition and requires guidance to understand the new units and actions involved in multiplicative situations. Focusing on the operation of multiplying two numbers or memorizing multiplication facts before developing an understanding of multiplicative situations and their quantities prematurely narrows students' focus and gives students the wrong impression about the need to understand what it means to multiply and the situations in which multiplying is the appropriate thing to do.

To be able to assess students' understandings of multiplication concepts, one needs to consider the specific details of what it means to understand multiplication, how these understandings can be easily and effectively demonstrated, and how to interpret students' performances as evidence of understanding.

Elements of Understand Multiplication?

Building an understanding of the concept of multiplication requires developing a language for thinking about and describing multiplicative situations involving equal groups of quantities. In the most basic form, a multiplicative situation describes A equal groups with B things in each group, where the total number of things is equal to A x B. From our review of the literature on understanding multiplication, we have focused on five basic and interconnected concepts: (a) quantity, (b) multiplicative problem situations, (c) equal groups, (d) units relevant to discrete multiplication, and (e) how multiplicative situations differ from additive situations. Most of these understandings can develop from experiences using counting and grouping strategies to solve contextualized problems in the early grades.

Understanding quantity. The meaning of quantity often gets overlooked in addition situations, but a thorough understanding of quantity provides an important foundation for understanding multiplication. A quantity is a characteristic of objects that can be counted or measured, and the value of a quantity consists of a *number* and a *unit* (Center for Research in Mathematics and Science Education, 1998). Twelve pennies is an example of a quantity—it includes both a number (12) and a unit (pennies). Number names (e.g., twelve) are often used to describe the number portion of a quantity. Other representations for the number part of a quantity include pictures (e.g., 12 circles representing 12 pennies) and the numerals 0-9 arranged in a base-10 place-value system (e.g., 12 representing twelve or 109 representing one hundred nine). In addition to the numbers in these examples, a unit must be specified to know the complete quantity. Neither the number twelve nor the numeral 12 tells *what* is being quantified. Although a pictorial representation of a quantity explicitly shows one possible unit, such as 12 circles, this unit may be representing a different unit, such as 12 pennies or 12 round cans that contain some quantity of yet another item. In each case, both the number and the units must be clearly represented in words, numbers, or pictures to completely specify the quantity.

Understanding multiplicative problem situations. Students need to have sufficient experience figuring out the meaning of word problems describing multiplicative situations to make sense of those situations and to distinguish them from other situations suggesting addition, subtraction, or division operations. Students also need to understand the relationships between multiplication and division and be able to find each of the three possible unknown quantities in grouping/partitioning situations (e.g., given 24 cookies arranged in four bags of six cookies each, three different problems can be posed by providing any two of these three pieces of information and asking for the third). Carpenter, Fennema, Franke, Levi, & Empson (1999) refer to these three problem types as multiplication (the number of groups and the number in each group are known but the total is unknown), measurement division (the total and the number in the groups is known but the number of groups is unknown), and partitive division (the total and the number of groups is known but the number in each group is unknown). Meanings of conventional multiplication notation (e.g., $4 \times 6 = 24$) should be connected to the language and meanings of multiplicative situations and their units.

Understanding equal groups. Students need to have sufficient experience arranging objects into groups to understand the role of equal groups in multiplicative situations and to establish a motivation for multiplying equal groups instead of counting all of the objects in the problem. Number sense includes the ability to compose and decompose numbers. Multiplicative reasoning includes using factors and multiples as equal groups in composition and decomposition of numbers instead of using additive compositions. For example, six objects can be arranged into multiplicative groups (e.g., one group of six, two groups of three, three groups of two, or six groups of one) rather than additive groups (e.g., one and five, two and four, three and three, and six and zero). Visual images are particularly helpful in understanding grouping (e.g., the difference between a disorganized collection of 48 items and the same 48 items organized into 4 groups of 12 items or an array of 6 rows and 8 columns).

Understanding units relevant to multiplication. Students need to have sufficient experience with counting and arranging objects into groups to understand the differences between various kinds of units that are relevant to multiplication (as distinct from units that apply to additive situations), particularly the difference between singleton units (e.g., ones, donuts, or cents) and composite units (e.g., twos, fives, tens, dozens, or rows of x and columns of y in an x by y array).

Students need to understand that composite units can also be counted (e.g., the number 30 is three 10s as well as the counting number after 29).

Understanding the relationships between multiplication and addition. Addition most often involves the joining of unequal quantities of the same unit (e.g., adding 29 cents and 54 cents). However, the two numbers in a multiplication situation typically refer to different units (e.g., multiplying 29 cats by four legs for each cat) representing the number of equal groups and the number of items in each of the groups. Understanding the connections between multiplication and addition should include knowing how a product is related to a sum from the repeated addition of the number of items in the equal groups.

To develop flexibility in both additive and multiplicative reasoning, students need to have sufficient experience with counting, joining, and grouping processes to understand the differences involved in moving from addition to multiplication. Developing an understanding of the iterative process of multiplication can begin with skip counting or repeated addition (particularly with groups of 10), because these counting quantities themselves represent groupings that have equal numbers of the same units. Understanding the iterative process of multiplication at this informal level (number of groups and number in each group) can provide a foundation for understanding more formal definitions of various multiplicative structures.

Given these five elements of elementary multiplication, we can begin to think about assessing students' understanding of multiplication through developmentally appropriate tasks that are able to provide evidence of understanding of these interconnected concepts.

Assessment

For our assessment of children's understanding of multiplication, we started with a series of four different problems about 3×4 (see Table 1). These four problems were repeated three more times using increasing number combinations (5×8, 8×7, and 9×6). As shown in Table 1, these problems were intended to provide evidence of understanding in three different forms: words, pictures, and numbers.

Table 1. *Initial Series of Problems*

Problem #	Problem	Evidence Form
1.	$3 \times 4 =$ _____	Numbers
2.	Write a story problem for which 3×4 is the correct number sentence.	Words

3.	Draw a picture that shows 3 × 4.	Pictures
4.	Write an addition number sentence that shows 3 × 4.	Numbers

This series of problems addressed multiple goals for the assessment. We started with a conventional number sentence that we expected fourth-grade children would find familiar and for which they would quickly produce a correct answer. This problem attended to the goal of correct answers, whether through recall of facts, mental computations, or counting strategies. Next, we asked students to write a story problem for the number sentence to check their connections between multiplicative number sentences and problem situations, quantities, and units. These first two problems are similar to the approach taken by O'Brien and Casey (1983a), which was based on an earlier report by McIntosh (1979).

Next in this problem sequence, we asked students to draw a picture to allow us to examine their visual representations for multiplicative structures (most often shown as groupings with the same number of items in each group) and units, as distinct from those in additive relationships. We then asked students to write an addition number sentence that showed the same thing as the given multiplication number sentence to provide additional data about students' understandings of how multiplication and addition are related.

This collection of four problems created multiple opportunities for students to make connections between what they knew and the basic multiplication concepts (quantity, problem situations, grouping, units, and relationships with addition). Although the format of some of these problems might have been less familiar to students' whose curriculum consisted primarily of number facts, standard algorithms, and application word problems than to students experiencing the *Investigations* curriculum, this set of problems asked for a variety of representations to provide multiple opportunities to show understanding of basic multiplication concepts, particularly the most basic conception of multiplication as a number of equal groups. The absence of any evidence of these key ideas in students' responses would provide a compelling argument that they had not yet developed the desired basic understandings of multiplication.

We followed these 16 problems with 10 word problems to provide evidence of students' understanding of various multiplication and division situations and their functional facility in solving such word problems. We designed this collection of word problems using the various multiplication and division problem types identified in research on children's mathematical thinking (see Carpenter et al., 1999). We included multiplication, measurement division, and partitive division situations involving grouping/partitioning, rate, price, and multiplicative comparison. We used measurement division and partitive division problems in what was primarily a multiplication study to provide evidence of students' understandings of the structure and context of the problems and to illuminate careless decoding strategies that represent taking shortcuts in comprehending the words in the problem. The last four of these word problems used the same number combinations as the four multiplication number sentences we had given earlier in the interview (i.e., Items 1, 5, 9, and 13).

Data collected for this study included (a) students' written work created during the interview and (b) interviewer field notes about students' responses to probing questions. Probing questions included "How did you get that?" or "Can you tell me how your story problem [or picture, or number sentence] shows _______?" which the interviewer asked after students responded to each interview item.

Analysis of the data involved comparing correct answers within and across the two groups of students and analyzing students' written work and verbal comments during the interviews for evidence of understanding of the basic concepts of multiplication described above.

Results

Comparing Correct Answers

The comparative results for percent of correct answers on the interview problems are shown in Table 2. Also shown are the percents of immediate responses to the bare number sentences, defined as providing an answer within approximately 2 seconds (and attributed to recall of facts). Looking first at the results for the traditional group (Grade 4 students) we see that they immediately produced a correct answer 100% of the time only with 3×4. Only 70% of these students gave an immediate response to 5×8, and they provided fewer immediate responses and were less accurate as the products got larger (8×7 and 9×6), with only 20% immediate responses and 70% and 90% accuracy for the two largest products. In comparison, only 7% of the *Investigations* group (Grade 3 students) provided an immediate response to 3×4, and none of them gave an immediate response on the other three number sentences. However, this group was 100% accurate on all four of the bare number sentences.

Table 2. *Correct Answers by Group*

Bare Number Sentences and Conceptual Problems	**Grade 4 Traditional**		**Grade 3 *Investigations***	
	% Immediate Response	**% Correct Answer**	**% Immediate Responses**	**% Correct Answers**
1. 3 × 4 =	100	100	7	100
2. Write a story problem		10		73
3. Draw a picture		20		100
4. Write an addition number sentence		40		93
5. 5 × 8 =	70	100	0	100
6. Write a story problem		20		87
7. Draw a picture		40		100
8. Write an addition number sentence		40		100
9. 8 × 7 =	20	70	0	100
10. Write a story problem		20		93
11. Draw a picture		20		100
12. Write an addition number sentence		30		100
13. 9 × 6 =	20	90	0	100
14. Write a story problem		20		100
15. Draw a picture		20		100
16. Write an addition number sentence		40		100
Mean: Bare Number Sentences (Items 1, 5, 9, 13)	53	90	2	100
Mean: Conceptual Problems (Items 2-4, 6-8, 10-12, 14-16)		27		96
Mean: Word Problems (17-26)		84		96
Mean: All Items (1-26)		58		96

The traditional group struggled with the conceptual problems (write a story problem, draw a picture, and write an addition number sentence), and produced acceptable answers only 27% of the time. In contrast, the *Investigations* group produced acceptable answers on the conceptual problems 96% of the time. Interestingly, those students in the *Investigations* group who initially struggled with writing a story problem or an addition number sentence early in the interview performed better as the interview progressed. Generally, students in the traditional group who did not have a useful strategy for these conceptual problems early in the interview were unable to improve their performance as the interview progressed.

Table 2 also shows that the traditional students performed much better (84% correct) on the word problems (Items 17-26) than they did on the conceptual problems, although they still did not perform as well as the *Investigations* group (96% correct). Item 20 (a multiplicative comparison problem involving 3 × 6; *A giraffe is 3 times as tall as a man is. If the man is 6 feet tall, how tall is the giraffe?*) was the only item where the traditional group outperformed the *Investigations* group (100% versus 67%). Some of the students in the *Investigations* group struggled to make sense of the difficult structure of this problem, which asked them to identify the relation between two quantities where one quantity was a multiple of the other. This comparative relation is very different from the relation between a number of groups and the number of objects in each group (Carpenter et al., 1999). However, for the traditional group, this particular multiplicative comparison structure easily fit a key word strategy. All of the traditional students recognized the word *times* in the problem and successfully multiplied 3 times 6.

Table 3 compares means, modes, and standard deviations across the two groups of students for the three categories of items (bare number sentences, conceptual problems, and word problems), as well as for correct answers on all items. A one-tailed *t*-test of the differences in the means for the two groups of students in each of these categories shows that the correct answers for the *Investigations* group were significantly higher than for the traditional group only for the conceptual problems and that this difference in correct answers on the conceptual problems accounts for the significant difference in the total number of correct answers for these two groups.

Looking at scores and explanations for individual students, we noted that correct answers on this collection of word problems involved issues of understanding the multiplication and division situations as well as flexibility in problem solving strategies. Errors by students in the traditional group often resulted from not understanding the structure of the problem, not remembering the multiplication fact they needed, or retrieving an incorrect fact. When they could not correctly recall a particular multiplication fact, they did not attempt other, more reliable strategies for computing an answer to the problem. In particular, for Item 18 (a rate situation involving 21 ÷ 3; *Sarah walks 3 miles an hour. How long will it take her to walk 21 miles?*), traditional students' typical strategy of looking for key words and searching for an appropriate operation produced many errors.

Table 3. *Correct Answers Statistical Comparison*

	4 Bare Number Sentences	**2 Concept Problen**	**10 Word Problems**	**Total Correct Answers**
Traditional Group				
Mean	3.60	3.20	8.40	15.20
Mode	4.00	0.00	10.00	7.00
Standard Deviation	0.70	4.08	2.22	5.53
Investigations **Group**				
Mean	4.00	11.47	9.60	25.07
Mode	4.00	12.00	10.00	26.00
Standard Deviation	0.00	0.92	0.51	1.10
***t*-Test Results**				
Mean Diff (*Invest*-Trad)	0.40	8.27	1.20	9.87
t Stat	1.8091	6.3068	1.6796	5.5656
P(T<=t) one-tail	0.0519	0.0000	0.0620	0.0002

Analyzing Conceptual Problems for Evidence of Understanding

Analyzing students' work involves looking for evidence of understanding and misconceptions in the products they produced and in their explanations of the work. This analysis of students' work is fundamentally different from counting the number of correct answers and computing percentages. Examining details of what students produce (or what they say during conversations) provides insights into their understandings of key concepts that cannot be inferred from percentages of correct answers alone.

Writing a story problem. First, consider some examples of the ways these students responded to the request to "Write a story problem for which __ × __ is the correct number sentence." Story problems were acceptable if they described a given number of groups with a given number of items in each group, totaling to the appropriate number.

Traditional students often wrote story problems using additive structures that had the same answer as the multiplication number sentence or simply followed its language. These typically used the same unit for both quantities, which is another indication of an additive structure. For example, traditional students wrote these story problems for 3 × 4:

- "Sue had 4 candles and Tamara had 8. How many did they have in all?"
- "Bobby had 4 baseball cards. He got 3 times as many as he had already. How many did he have in all?"

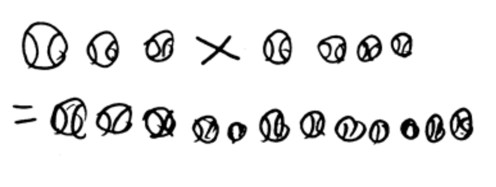
One traditional student wrote a story problem that began with an additive situation, recognized this was not correct, and then specified a multiplication operation: "Josh had 3 baseball cards and his friend had 4. ~~How many do they have in all?~~ How many would they have if they multiplided [sic] these numbers?"

Students in the *Investigations* group nearly always described a grouping situation, identified a number of groups, specified a number in each group, and asked for the total number of items. For example:

- "I had 4 boxes of doughnuts. Each had 3 doughnuts. How many doughnuts do I have?"
- "I have 5 fish. Each one gets 8 piecies [sic] of fish food. How many piecies [sic] do I have to have?"
- "I have 8 boxes of toys. Each box has 7 toys in it. How many toys do I have?"

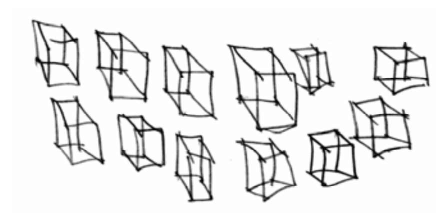
Drawing a picture. Traditional students' responses to the request to "Draw a picture that shows __ × __" provided evidence of the same misconceptions about multiplication as their word problems: (a) multiplication is the structure of the multiplication number sentence, or (b) multiplication is the answer to a multiplication number sentence. Figure 1 shows four of these responses. Picture A shows a multiplication number sentence with a tree and grass added. Picture B shows baseballs in place of the numbers in a horizontal number sentence. Picture C shows stars in place of numbers in a vertical number sentence. Picture D shows the answer to 3 × 4 as 12 cubes without any grouping of the cubes.

Picture A



Picture B

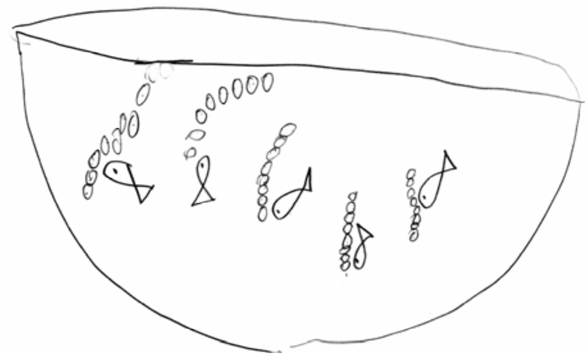
Picture C



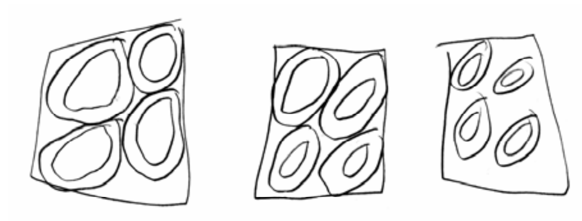
Picture D

Figure 1. *Pictures by Grade 4 Students in the Traditional Group*

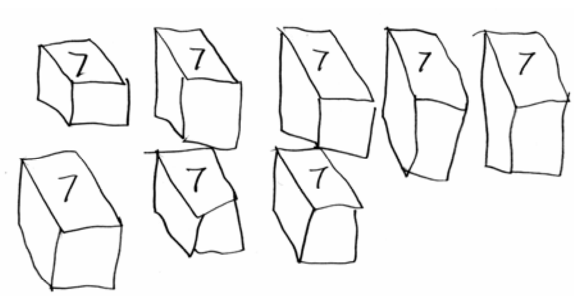
Figure 2 shows four examples that are typical of pictures by students in the *Investigations* group. Picture E shows five fish in a fish bowl, with each fish having eight pieces of fish food. Picture F shows 12 donuts arranged in three boxes of four donuts each. Picture G shows eight boxes labeled as having seven items in each box. Picture H shows 56 squares arranged in an array of eight rows and seven columns.



Picture E



Picture F



Picture G

Picture H

Figure 2. *Pictures by Grade 3 Students in the Investigations Group*

In every case, the pictures of students in the *Investigations* group showed understanding of multiplication as grouping, and their pictures represented the number of groups, the number in each group, and the product of the multiplication. With few exceptions, the pictures of students in the traditional group did not indicate an understanding of multiplication as grouping. Instead, their meaning of multiplication was limited to the number sentence and the answer. Those few who wrote word problems about equal groups also created pictures of equal groups.

Writing an addition number sentence. Many students in the traditional group wrote addition number sentences that totaled to the same sum as the product of the multiplication but did not include addends that were related in any way to a multiplication situation. For example, their responses to “Write an addition number sentence that shows 3 × 4” included the following:

- three times four 3 × 4 = 12
- 6 + 6 = 12
- 3 + 4 + 1 + 4 = 12
- 3 + 4 = 7

These students’ responses indicate that they did not understand enough about how multiplication is different from addition to see how the question was asking for more than the same answer. They knew how to write an addition number sentence, but did not indicate an understanding that both addition and multiplication number sentences can be used to show a number of groups with the same quantity in each group.

In contrast, nearly all of the students in the *Investigations* group wrote either 4 + 4 + 4 = 12 or 3 + 3 + 3 + 3 = 12 (or both), indicating a clear understanding of the applicability of an addition process to represent a situation involving equal groups. These responses most likely reflected these students’ experiences using repeated addition as a strategy for determining answers to multiplication problems. These differences in responses make clear that there can be differences between students’ understandings of the structure of a problem and their choices of strategies for solving the problem.

Conclusions

The Smith and Smith (1996) study shows that the collection of tasks used provided meaningful evidence for assessing children’s understandings of multiplication concepts, including understandings of the relationships between multiplication and addition. The study also provides evidence that memorizing multiplication facts produced much less understanding of the basic concepts of multiplication in a group of fourth-grade students receiving traditional instruction than a standards-based curriculum and instruction produced among a group of younger third-grade students. This is consistent with the broader claim that a focus on computational skills alone works against the development of the view that learning mathematics is a sense-making activity (Robinson, Robinson, & Maceli, 2000). These results also show under what curricular circumstances students have the opportunity to develop robust understandings of basic multiplication concepts, which contrasts with the findings of O'Brien and Casey (1983a, b) for students experiencing a traditional computation focus.

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Change and Relationships in Elementary Preservice Teachers' Mathematics Pedagogical Beliefs, Teaching Efficacy Beliefs, and Content Knowledge

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Abstract

This study investigated the mathematics beliefs and content knowledge of 103 elementary preservice teachers in a developmental teacher preparation program that included a two course mathematics methods sequence. Preservice teachers' pedagogical beliefs became more cognitively-oriented during the teacher preparation program with these changes occurring during the two methods courses. Pedagogical beliefs remained stable during student teaching. The preservice teachers also significantly increased their personal efficacy for teaching mathematics throughout the program with these shifts occurring across both methods courses and into student teaching. Pedagogical beliefs and teaching efficacy beliefs were not related at the beginning of the program, but, in general, were positively related throughout the program. In addition, the preservice teachers' pedagogical beliefs were positively related to their specialized content knowledge for teaching mathematics at the end of the program.

Change and Relationships in Elementary Preservice Teachers' Mathematics Pedagogical Beliefs, Teaching Efficacy Beliefs, and Content Knowledge

Introduction

In mathematics education it is not uncommon for beginning preservice teachers to come to their teacher preparation programs with a traditional view of what it means to know and do mathematics: a view of mathematics as a fixed body of knowledge to be delivered to children, usually through clear, organized presentations and lectures. In contrast, current university mathematics education programs are more likely to advocate a constructivist view of teaching and learning such as that supported by the National Council of Teachers of Mathematics (2000). For preservice teachers to be successful within a reform program, they need to do more than learn methods and materials for teaching mathematics: they need to *change* their beliefs (Richardson & Placier, 2001).

Beliefs influence teacher behavior and decision-making (Thompson, 1992; Wilson & Cooney, 2002) and change in beliefs is a crucial precursor to real change in teaching. This change is not easy. It takes time for preservice teachers to align their pedagogical beliefs with current thinking on teaching and learning mathematics and to increase their efficacy for teaching mathematics. It is a difficult process requiring thoughtful reflection and examination of teaching and learning.

But altering pedagogical beliefs and teaching efficacy beliefs is only part of preparing preservice teachers to teach mathematics. For some years now mathematicians and mathematics educators have agreed that there is a need for strong mathematical content knowledge for elementary teachers. A small number of mathematicians and many more mathematics educators propose that there is *specialized content knowledge* (SCK) needed for teaching elementary mathematics that is unique from the common mathematical content knowledge necessary to be a functioning adult (Ball, Hill, & Bass, 2005). For example, most adults could easily compute the problem $12 \div 3$. However, few would recognize that the problem can be modeled as either 4 groups of 3 or 3 groups of 4, depending on the question asked. Knowing the difference between distributive and subtractive situations in division and being able to represent those situations through real world applications is representative of the specialized knowledge needed by elementary teachers.

Prior research on preservice teachers has mostly examined mathematics pedagogical beliefs, teaching efficacy beliefs, and content knowledge as individual constructs, typically in the context of a single course (Hart, 2002; Wilkins & Brand, 2004). Of further interest is understanding changes in these constructs over time as well understanding the relationships among these constructs.

Background

A recent mandate from the Board of Regents of Georgia required an increase in the number of upper-division mathematics courses for elementary preservice teachers. Prior to the mandate, students within our program took two mathematics courses and two mathematics methods courses. In response to the directive, the program was revised to contain four mathematics courses and one methods course.

In an effort to document the impact of the change on elementary preservice teachers at Georgia State, we began a longitudinal research effort we call the Mathematics Education Research Project (MERP). The project examines how these programmatic changes influence preservice teachers' mathematics pedagogical beliefs, teaching efficacy beliefs, and specialized content knowledge for teaching mathematics. This manuscript reports results from the preservice teachers in the program prior to the mandated changes and focuses on change in beliefs and the relationships between beliefs and specialized content knowledge. This comparative group provides data for future research on the effects of the new program and some guidance on what should be emphasized in content and methods coursework.

Related Research

A review of the body of research on teachers' mathematics pedagogical beliefs, teaching efficacy beliefs, and content knowledge highlights the importance of these constructs. To lay a foundation for the research in this paper, we will examine relevant findings in each area.

Teacher Pedagogical Beliefs

Research has shown that beliefs develop over time (Richardson, 1996); that they are well-established by the time a student enters college (Pajares, 1992); and that they develop during what Lortie (1975) terms the *apprenticeship of observation* which occurs over an individual's years as a student. Elementary teacher preparation programs have a limited amount of time to impact change in preservice teacher pedagogical beliefs—usually two years or less. Impacting change in pedagogical beliefs in mathematics may be limited to one course as is evidenced by the number of studies that look at change over one course or semester. Some earlier studies did not achieve the desired effect (Ball, 1989; Simon & Mazza, 1993), while more current studies did (Hart, 2002; Lubinski & Otto, 2004; Spielman & Lloyd, 2004; Wilkins & Brand, 2004).

While this snapshot work makes important contributions to the body of knowledge on pedagogical beliefs, it is also important to look at what happens to these beliefs over time. Vacc and Bright (1999) administered a pedagogical belief survey to 34 preservice teachers four times over a two-year period during a teacher preparation program, finding little change during the first year, but substantial change during the second year, reinforcing the need to study beliefs over time.

Teaching Efficacy Beliefs

Research establishes a robust relationship between a teacher's sense of efficacy and instructional strategies in the classroom (Riggs & Enochs, 1990) and shows that this relationship influences student achievement (Anderson, Green, & Loewen, 1988). The majority of the studies looked at *generalized* teaching efficacy, seeing it as a two-dimensional construct (Enochs, Smith, & Huinker, 2000) within Bandura's (1977) theoretical framework. The first dimension, *personal teaching efficacy*, represents a teacher's belief in his or her skills and abilities to be an effective teacher. The second dimension, *teaching outcome expectancy*, is a teacher's belief that effective teaching can bring about student learning regardless of external factors such as home environment, family background, and parental influences.

Hoy (2004) suggests that Bandura's theory implies that efficacy beliefs may be most malleable early in learning, making the first few years of teacher development critical to the long-term development of teaching efficacy. Once teaching efficacy beliefs are established, they are highly resistant to change; but studies suggest that coursework and the student teaching experience have differential impacts upon the two dimensions. Personal teaching efficacy increases during coursework and continues to increase during the student teaching experience (Hoy & Woolfolk, 1990; Plourde, 2002). However, teaching outcome expectancy beliefs increase during college coursework but decline during student teaching. This decline has been attributed to the unrealistic optimism preservice teachers have prior to student teaching about teachers' abilities to overcome negative influences (Hoy & Woolfolk, 1990).

Although there are numerous studies on generalized teaching efficacy, there has been less research specifically on the mathematics teaching efficacy of elementary preservice teachers. Those that did look at mathematics examined the effect of a single mathematics methods course and indicated significant increases in mathematics teaching efficacy upon completion of the course (Huinker & Madison, 1997; Utley, Moseley, & Bryant, 2005).

Content Knowledge

The scholarly literature on teachers' mathematical content knowledge can be traced back for several decades. General themes are observable and contribute to what we know and do not know about elementary teachers' content knowledge and in some cases its relationship to student learning. Mewborn (2001) and Ball, Lubienski, and Mewborn (2001) provide recent summaries of research on mathematical content knowledge in the preparation and teaching practice of K-8 teachers, updating and expanding an earlier review by Fennema and Franke (1992). Mewborn identifies several themes that emerged over the last 40 years concluding that there is little correlation between the number of higher mathematics courses a teacher takes and student learning (Ball, 1990, 1991; Begle, 1972; Eisenberg, 1977), that there are certain domains within mathematics where many teachers do not have deep conceptual understandings (Post, Harel, Behr, & Lesh, 1991; Tirosh, Fischbein, Graeber, & Wilson, 1999), and that most elementary teachers in the United States do not have a deep understanding of the mathematics they teach (Bransford, Brown, & Cocking, 2001; Committee on Science and Mathematics Teacher Preparation, 2001; Mathematics Teacher Preparation Content Workshop Steering Committee, 2001).

The question as to the nature of the mathematical knowledge needed by teachers and the remedy for the problem of obtaining that knowledge remains an issue. Some of the most extensive work in this area comes from Hill, Schillings, and Ball (2004). They examined both the nature of mathematical knowledge needed to teach and the relationship between this knowledge and student learning. Taking a multidimensional approach to measuring content knowledge, they argued that although much has been done to research teachers' content knowledge, further work is needed to precisely map the knowledge needed for *teaching* mathematics. In developing their Learning Mathematics for Teaching (LMT) (Hill, Schillings, & Ball, 2004) instruments, they have made progress in using test items designed to identify specific knowledge and reasoning that is important for teaching mathematics from a reform perspective including generating representations, interpreting student work, and analyzing student mistakes.

Research Questions

In the Mathematics Education Research Project (MERP) we examine three constructs: mathematics pedagogical beliefs, teaching efficacy beliefs, and specialized content knowledge. We ask:

- How do elementary preservice teachers' mathematics pedagogical beliefs and teaching efficacy beliefs change during a teacher preparation program?
- What is the relationship between elementary preservice teachers' mathematics pedagogical beliefs and teaching efficacy beliefs during a teacher preparation program?
- What is the relationship between elementary preservice teachers' mathematics beliefs and their specialized content knowledge for teaching mathematics at the end of a teacher preparation program?

Methodology

Participants and Setting

The participants in this study were elementary preservice teachers enrolled in a two-year undergraduate teacher education program at Georgia State University. A total of five cohorts of students (n = 103) are included in our results. Students within a cohort are admitted concurrently and complete all education courses together. The old program consisted of four semesters of coursework which included two mathematics methods courses taught in consecutive semesters. Each of the first three semesters included two-day-a-week field placements followed by a semester of student teaching. The field placements and coursework followed a developmental model with preservice teachers starting their placements in pre-kindergarten and finishing in fifth grade prior to student teaching. Other mathematics requirements in the program included two mathematics content courses for teachers taught through the mathematics department in addition to any university requisite mathematics coursework.

The mathematics methods courses were taught by faculty in the elementary education department who share a common philosophical orientation toward the teaching and learning of mathematics. Thus across the courses the focus was consistent with the constructivist paradigm espoused by the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000) in that all students should learn important mathematical concepts and processes with understanding. The preservice teachers were exposed to the features of a *Standards-Based Learning Environment* with a focus on the processes of problem solving, representations, communication, connections, and reasoning and proof. Important goals of the courses included developing (a) beliefs consistent with the perspective of the *Principles and Standards*, (b) understanding of children's thinking about important mathematics concepts, (c) abilities to create problem-solving learning environments for children to facilitate discourse and understanding, and (d) abilities and confidence as a lifelong learner and doer of mathematics. The first methods course focuses on content and pedagogy for pre-kindergarten through second grade students with field placements in those grades. The second course emphasizes third through fifth grades with corresponding field placements.

Instrumentation

Two instruments, the Mathematics Beliefs Instrument (MBI) and the Mathematics Teaching Efficacy Beliefs Instrument (MTEBI), were administered to the participants four times during the teacher preparation program. In addition, the Learning Mathematics for Teaching Instrument (LMT) was administered at the end of student teaching.

The MBI is a 48-item Likert scale instrument designed to assess preservice teachers' beliefs about the teaching and learning of mathematics and the degree to which these beliefs are cognitively aligned (Peterson, Fennema, Carpenter, & Loef, 1989, as modified by the Cognitively Guided Instruction Project). The three subscales include: (a) relationship between skills and understanding (CURRICULUM), (b) role of the learner (LEARNER), and (c) role of the teacher (TEACHER). The 16 item CURRICULUM subscale examines the degree to which teachers believe that mathematics skills should be taught in relation to understanding and problem solving. The LEARNER subscale contains 15 items that assess the degree to which teachers believe that children can construct their own mathematical knowledge. The 17 items on the TEACHER subscale address the extent to which teachers believe that mathematics instruction should be organized to facilitate children's construction of knowledge. The instrument uses a Likert scale with five response categories (strongly agree, agree, uncertain, disagree, and strongly disagree) with higher scores indicating beliefs that are more cognitively-aligned. These subscales have high reliability (Chronbach's alpha = .80 for CURRICULUM, .89 for LEARNER, and .90 for TEACHER) and represent independent constructs based on confirmatory factor analysis.

The MTEBI consists of 21 items, 13 on the Personal Mathematics Teaching Efficacy (PMTE) subscale and 8 on the Mathematics Teaching Outcome Expectancy (MTOE) subscale (Enochs, Smith, & Huinker, 2000). The two subscales are consistent with the two-dimensional aspect of teaching efficacy. The PMTE subscale addresses the preservice teachers' beliefs in their individual capabilities to be effective mathematics teachers. The MTOE subscale addresses the preservice teachers' beliefs that effective teaching of mathematics can bring about student learning regardless of external factors. The instrument uses a Likert scale with five response categories (strongly agree, agree, uncertain, disagree, and strongly disagree) with higher scores indicating greater teaching efficacy. These subscales have high reliability (Chronbach's alpha = .88 for PMTE and .81 for MTOE) and represent independent constructs based on confirmatory factor analysis.

The LMT examines teachers' specialized content knowledge for teaching mathematics (Hill, Schilling, & Ball, 2004). The instrument assesses this knowledge by posing mathematical tasks that reflect what teachers encounter in the classroom such as assessing students' work, representing mathematics ideas and operations, and explaining mathematical rules or procedures. Content knowledge subscales in this instrument include: (a) number and operations, (b) patterns, functions, and algebra, and (c) geometry (Hill & Ball, 2004). Content validity was established by mapping items for congruence with the National Council of Teachers of Mathematics Standards (Siedel & Hill, 2003). Analysis of reliability indicated alpha coefficients of .79 for the number and operations subscale, .75 for the patterns, functions, and algebra subscale, and .85 for the geometry subscale (G. Phelps, personal communication, October 6, 2006).

Table 1 shows points of data collection for instruments, the sequence and length of placements, and when the mathematics methods courses were completed.

Table 1

Sequence of Teacher Preparation Program and Data Collection

	Semester 1	Semester 2*	Semester 3*	Semester 4*
Mathematics methods courses	None	cus on PreK-2 mathemat	cus on 3-5 mathemat	None
Field Placements	PreK – 5 weeks K – 9 weeks	lst – 7 weeks 2nd or 3rd – 7 weeks	4th – 7 weeks 5th – 7 weeks	Student teaching
Administration of MBI & MTEBI (Four times)	None	**(1) INITIAL** Week 1 **(2) POST 1** Week 14	**(3) POST 2** Week 14	**(4) FINAL** Week 14
Administration of LMT (one time)	None	None	None	**(4) FINAL** Week 14

*Asterisks denote semesters included in this study.

Results

Mean scores and standard deviations across the administrations of the MBI subscales and MTEBI subscales are provided in Table 2. Table 3 indicates the statistical significance of the differences between these means using Wilks' Lambda and its associated F-statistic.

Table 2
*Means and Standard Deviations for Pedagogical Beliefs and Mathematics Teaching Efficacy Scores**

	Means				Standard deviations			
Subscale	Initial	Post 1	Post 2	Final	Initial	Post 1	Post 2	Final
CURRICULUM	3.04	3.25	3.33	3.34	.32	.47	.43	.49
LEARNER	3.08	3.38	3.55	3.55	.43	.49	.49	.60
TEACHER	3.31	3.60	3.71	3.77	.39	.45	.53	.58
PMTE	3.54	3.71	3.94	4.18	.56	.56	.56	.65
MTOE	3.44	3.50	3.63	3.64	.40	.43	.50	.51

* Represented on a five point Likert scale

As indicated in Table 3, the preservice teachers had significant increases in overall CURRICULUM, LEARNER, and TEACHER subscales scores. The preservice teachers' beliefs became more cognitively-aligned during the teacher preparation program with these significant changes occurring across the semesters they were enrolled in the two methods courses with the exception of the CURRICULUM subscale during the semester of the second methods course. During student teaching the scores on these three subscales remained largely unchanged.

Table 3
*F-Values (p-values) for Pedagogical Beliefs and Mathematics Teaching Efficacy Scores**

Subscale	Overall	Initial to Post 1	Post 1 to Post 2	Post 2 to Final
CURRICULUM	19.35 (.000)	29.28 (.000)	3.13 (.080)	.138 (.711)
LEARNER	37.76 (.000)	44.74 (.000)	13.14 (.000)	.002 (.968)
TEACHER	33.37 (.000)	55.86 (.000)	4.77 (.031)	1.20 (.276)
PMTE	26.22 (.000)	11.02 (.001)	20.90 (.000)	10.51 (.002)
MTOE	6.88 (.000)	1.83 (.179)	6.04 (.016)	.02 (.883)

*For the overall comparisons, df = 3, 100; for all other comparisons, df = 1, 102

Data from the PMTE subscale revealed the preservice teachers had significant increases in their overall personal efficacy for teaching mathematics (see Table 3). These significant mean increases in scores occurred consistently throughout the program across the semesters of both methods courses and into student teaching. MTOE subscale scores also showed significant increases during the teacher preparation program. The preservice teachers' outcome expectancy beliefs significantly increased with this change largely occurring during the semester of the second methods course and these beliefs remaining essentially constant during student teaching.

Correlations between teaching efficacy beliefs and pedagogical beliefs across the administrations are provided in Table 4. At the beginning of the semester of the first methods course, there were no significant relationships between PMTE and MTOE subscale scores and CURRICULUM, LEARNER, and TEACHER subscale scores. However, at the end of the semesters after the first and second methods courses as well as after student teaching the PMTE and CURRICULUM, LEARNER, and TEACHER subscale scores were positively related with slight to moderate correlations. In addition, in general, the MTOE and CURRICULUM, LEARNER, and TEACHER subscale scores had slight, positive relationships at the end of the semesters of the first and second methods course as well as student teaching.

The results of the correlation analysis also revealed some significant relationships between specialized content knowledge and beliefs at the end of student teaching (see Table 4). The preservice teachers' scores on the LMT were slightly, positively related to CURRICULUM and LEARNER subscale scores. However, there were no relationships between the LMT scores and TEACHER, PMTE, and MTOE subscale scores.

Table 4

*Pearson Product Moment Correlations between Pedagogical Beliefs, Mathematics Teaching Efficacy, and Specialized Content Knowledge Scores**

	PMTE				MTOE				LMT
Subscale	Initial	Post 1	Post 2	Final	Initial	Post 1	Post 2	Final	Final
CURRICULUM	-.018	.289**	.389**	.380**	.020	.175	.292**	.336**	.238**
LEARNER	.098	.257**	.377**	.452**	-.088	.216**	.363**	.303**	.224**
TEACHER	.004	.380**	.429**	.593**	-.040	.257**	.367**	.320**	.127
PMTE									.058
MTOE									-.169

*n = 103

**Correlation is significant with $p < .05$

Discussion

This study examined three constructs during a teacher preparation program: pedagogical beliefs, teaching efficacy beliefs, and specialized content knowledge for teaching mathematics. Over one hundred preservice teachers in five different cohorts were studied during the two years of their teacher preparation program. The program took a constructivist approach to mathematics teaching and learning, attempting to provide experiences for students that encouraged alignment of beliefs with current reform recommendations. The students experienced a two-course mathematics methods sequence that aligned developmentally with their grade level field placements. Our results are interpreted within the framework of this program.

Changes in Beliefs

Our first research question focused on *changes in beliefs* during the program. The preservice teachers' mathematics pedagogical beliefs about the relationship between skills and understanding (CURRICULUM subscale) and the role of the teacher (TEACHER subscale) and learner (LEARNER subscale) became more cognitively-aligned during the teacher preparation program. The significant shifts occurred across the semesters they were enrolled in the two methods courses, with one exception. The subscale which measures beliefs that skills should be taught in relation to understanding (CURRICULUM) did significantly increase during the first methods course but not during the second methods course. During student teaching the preservice teachers' pedagogical beliefs as measured by the three subscales remained stable.

Overall, increased alignment of pedagogical beliefs with a cognitive orientation and hence a reform perspective during the two-semester methods sequence is consistent with earlier research on change during methods courses (Author, 2002; Lubinski & Otto, 2004; Spielman & Lloyd, 2004; Wilkins & Brand, 2004). The methods coursework immersed students in a reform perspective, and they were given opportunities to experience success in implementing reform practices in their field placements. They were supported by mathematics education faculty and frequently saw the benefits of a constructivist perspective to teaching and learning.

Some of the results from our question on change are of particular interest. First, after a significant change in preservice teachers' beliefs that skills should be taught through problem solving and with understanding (CURRICULUM subscale) during the first methods course these beliefs did not significantly increase during the second methods course. Since the academic focus during the second methods course is grades 3, 4, and 5, it is hypothesized that this experience had a differential effect on beliefs about curriculum implementation. The preservice teachers may have found less alignment of practices in their field placements with more emphasis on the mastery of individual skills in isolation than demonstrated in their methods course, resulting in a leveling of the belief that skills should be taught in relation to understanding.

A second interesting finding related to change in beliefs is that all three pedagogical beliefs subscale scores essentially remained the same during student teaching. The maintenance of the preservice teachers' beliefs during student teaching is promising, since the more traditional practices often experienced in the field can be counter intuitive to beliefs learned during university experiences. Maintenance of cognitively-aligned pedagogical beliefs provides optimism that those beliefs may be more secure and carried forward into the actual classroom decision-making of these future teachers. The particular programmatic features experienced by the preservice teachers, including the two course mathematics methods sequence and time-intensive, developmental field placements, seemed to have established these cognitively-oriented beliefs and allowed for stabilization during student teaching.

Across the program the preservice teachers consistently developed stronger beliefs in their skills and abilities to teach mathematics effectively as indicated by the PMTE subscale. The support provided by the two methods courses as well their experiences in student teaching not only sustained but increased their personal teaching efficacy beliefs. The preservice teachers were exposed to what Bandura (1977) has emphasized as two important sources of efficacy beliefs: mastery and vicarious experiences. The preservice teachers had significant field experiences (2 days per week) during both methods courses and complete immersion in the field during student teaching; this gave ample opportunity for successful mastery experiences in teaching mathematics. Furthermore, the preservice teachers were exposed to successful models of mathematics instruction from the instructors of the methods courses and some of the cooperating teachers in the field placements.

Outcome expectancy beliefs, or the preservice teachers' beliefs that effective teaching of mathematics can bring about student learning regardless of external factors, also increased during the program as indicated by the MTOE subscale. This increase largely occurred in the second methods course with the beliefs remaining essentially the same during student teaching. Earlier studies indicate that preservice teachers' outcome expectancy beliefs significantly decline during student teaching (Hoy & Woolfolk, 1990). This decrease in beliefs is attributed to the unrealistic optimism of preservice teachers toward impacting student learning prior to the immersing student teaching experience. Perhaps the substantial field experiences of the preservice teachers in this study prior to student teaching somewhat tempered this expectation and contributed to this positive finding in that these beliefs remained consistent during student teaching.

Interrelatedness of Beliefs

Our second research question examined the *interrelatedness of beliefs* throughout the program. At the beginning of the first methods course, there were no significant relationships between teaching efficacy beliefs (PMTE and MTOE subscales) and pedagogical beliefs (CURRICULUM, LEARNER, and TEACHER subscales). Given the emphasis in the methods courses on teaching from a reform perspective, it is not surprising to find that there were, in general, positive relationships after the first and second methods courses and student teaching. Throughout the program, the preservice teachers who had stronger beliefs in their skills and abilities to teach mathematics effectively generally had more cognitively-oriented beliefs toward the teaching and learning of mathematics. In addition, the preservice teachers who believed more strongly that effective teaching of mathematics could bring about student learning regardless of external factors generally held more cognitively-oriented beliefs toward mathematics instruction.

Interrelatedness of Beliefs and Specialized Content Knowledge

Our third research question addressed the *interrelatedness of beliefs and specialized content knowledge* after the program. The preservice teachers that had more specialized content knowledge for teaching mathematics (LMT) were more likely to believe that children can construct their own mathematical knowledge (LEARNER subscale) and that mathematics skills should be taught with understanding (CURRICULUM subscale). However, there were no relationships between the specialized knowledge for teaching mathematics (LMT) and beliefs that mathematics instruction should be organized to facilitate children's construction of knowledge (TEACHER), beliefs toward personal teaching efficacy (PMTE subscale), and teaching outcome expectancy (MTOE subscale). What is interesting in these results is the disconnect between preservice teachers' specialized content knowledge and their belief in their skills and abilities to teach mathematics effectively. It appears that preservice teachers can be quite efficacious about their teaching and not have developed strong specialized content knowledge for teaching mathematics. This naïve perspective is not surprising and is consistent with the human condition of not being aware of what you do not know.

Conclusions

Although preservice teachers enter teacher preparation programs with relatively well-entrenched beliefs about mathematics teaching and learning (Pajares, 1992), our results suggest that programs can have an impact on those beliefs. Consistent with other research, we found that during their coursework, preservice teachers developed beliefs more consistent with a reform perspective and became more efficacious about their skills and abilities to teach mathematics effectively and to influence student learning. Even during student teaching, personal teaching efficacy continued to increase while teaching outcome expectancy and pedagogical beliefs remained stable. It is optimistic that this enculturation experience in the schools did not undermine teacher change. The stability of these beliefs during student teaching seems to suggest that the distinctive features of the teacher preparation program, including two semesters of mathematics methods and time-intensive, developmental field placements, helped in developing well-established beliefs.

The pattern of our results is consistent with the view that both teaching efficacy and pedagogical beliefs are comprised of multiple constructs. We have added to this literature by showing that beliefs about the role of the learner, the teacher, and the relationship between skills and understanding in mathematics as well as personal teaching efficacy and teaching outcome expectancy vary over time and interact in different ways with each other and with other factors that influence mathematics teaching. One manifestation of this multi-dimensional aspect of beliefs is preservice teachers' *relative* resistance to change in their beliefs about teaching outcome expectancy and the relationship between skills and understanding in mathematics when compared to their personal teaching efficacy and beliefs about the roles of the teacher and learner. These two belief constructs were the only ones that did not consistently and significantly increase across the two methods courses. They also remained largely unchanged during student teaching.

In considering the findings of our study, we obviously cannot assert that the changes that occurred over the three semesters in our program will continue as these preservice teachers enter into their own classrooms; that is for another study. Also, we cannot confirm that the beliefs that they espouse will manifest themselves in classroom decision-making and practices with their own students. The conflict between espoused beliefs and beliefs in practice is a viable concern (Wilson & Cooney, 2002). Teachers who continue to hold reform beliefs are often hesitant to implement them within the culture of a traditional school setting (Hart, 2004). However, by carefully examining the process of change during the two years in a teacher preparation program and studying the interaction of the constructs that affect change, we are better informed about how to construct our program, which may assure more lasting change as preservice teachers make their way into their professional careers.

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The Kennesaw State University Mathematics Methods Model

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Abstract

Kennesaw State University's comprehensive, nine-credit-hour, methods course integrates general and mathematics-specific pedagogical training with a structured four-week field experience prior to student teaching. This course blends essential units on conceptual understanding of mathematics, lesson planning, assessment, classroom management, and diversity with mathematics-specific methods. All topics are aligned with National Council of Teachers of Mathematics standards and Georgia Performance Standards. Throughout the course, students complete a variety of assignments that require them to practice the skills highlighted in class readings and discussions, and they adapt and generalize those skills during their field experiences. Students have numerous opportunities in class and in the field to implement and to reflect upon pedagogical and assessment strategies and to receive feedback from course instructors, from other students, and from collaborating teachers. This intense course has many benefits and challenges for both the instructors and the students, but it is one of the most highly anticipated courses of secondary mathematics education majors at Kennesaw State University. With strong support of department administrators and the entire mathematics education faculty, this methods class has been quite successful in preparing the seniors for their student teaching experiences.

The Kennesaw State University Mathematics Methods Model

Course Overview

At Kennesaw State University, all secondary mathematics education majors enroll in a comprehensive nine-credit-hour methods course, entitled "Teaching of Specific Subject" (TOSS), usually in the semester prior to student teaching. This course includes twelve weeks of intensive classroom instruction, with three three-hour class meetings each week (usually, Monday, Wednesday, and Friday mornings). In the middle of the semester, TOSS students complete a four-week, half-day field experience during which class sessions are suspended. At their assigned middle or high schools, the students observe classes and various school functions and teach a unit, of their own design, under the guidance of a collaborating teacher and university supervisor. TOSS instructors serve as university supervisors, but, with large classes, additional mathematics education faculty assist with observations. Students earn six credit hours for successful completion of the classroom portion of the course and three credit hours for a successful field experience. This credit allocation enables students to repeat only the field experience if they were unsuccessful in that aspect of the course but were successful in the classroom portion.

In order to qualify for acceptance to TOSS, candidates complete an application process under the guidance of their academic advisors during the semester prior to enrollment. Prerequisites for acceptance into the course include admission to teacher education through the Bagwell College of Education and successful completion of two courses, an education course entitled "Learning, Motivation, and Classroom Management" and a mathematics course entitled "Advanced Perspectives in School Mathematics I."

Course Units and Assignments

The TOSS course includes five primary units designed to help students develop teaching skills that they can apply in their first teaching experiences. Joyce and Showers (1995) suggest that teachers can develop new skills through a process that closely integrates theory and practice. This process includes the development of the theory and rationale behind the skill, the demonstration of the skill, an opportunity to practice and to receive feedback on the skill, and the adaptation and generalization of the skill through sustained coaching. In order to help prospective teachers develop new skills, TOSS instructors compile a coursepack of readings that introduce students to the theories behind the teaching skills related to each unit of study. Instructors then facilitate classroom discussions that address the theories, the associated teaching skills, and the benefits and challenges of implementing the teaching skills. They also design classroom activities through which they demonstrate skills. Students complete assignments that enable them to practice their teaching skills and to receive extensive feedback from their instructors and from their classmates. The TOSS instructors then coach the students in their skills throughout their field experiences, and other mathematics education professors continue to coach students throughout their subsequent student teaching experiences.

In addition to the five major TOSS units, instructors integrate two ongoing content strands throughout the course. First, students read and discuss the Georgia Performance Standards (Georgia Department of Education [GADOE], n.d.) during the first week of class, and these standards are discussed in relation to every topic throughout the remainder of the course. Also, each week, instructors integrate examples of various mathematics teaching methods and strategies, as endorsed by NCTM (2000) and GADOE (n.d.), into the class. Such lessons include data collection, analysis, and regression activities that require the use of graphing calculators, calculator-based laboratories, and/or computer software; exploration of fractions and geometry with manipulatives; and problem solving activities that require cooperative learning.

The first TOSS unit centers around conceptual understanding of mathematics or learning mathematics with understanding, as promoted by NCTM (2000) and many mathematics educators (e.g., Brownell, 1935; Hiebert & Lefevre, 1986; Skemp, 1976). This unit actually begins prior to the start of the course with a pre-semester writing assignment that requires students to read much of *Principles and Standards for School Mathematics* and to reflect on the vision of mathematics education presented by NCTM (2000). During the first week of class, students read an article that describes the differences between conceptual and procedural knowledge (Skemp, 1976) as well as a case study of a student who has developed only procedural knowledge of mathematics (Erlwanger, 1973). In class, instructors and students discuss the readings and view film clips of teachers who are implementing lessons designed to promote conceptual understanding of various mathematics topics. Students work on problems that challenge their understanding of mathematics from a conceptual perspective. Students also search on-line sites for lesson plans that promote conceptual understanding of mathematics for various grade levels. Through these activities, students begin to develop skills in identifying what types of classroom instructional strategies promote connection-making in mathematics.

After students have committed to promoting conceptual understanding in their own teaching, they begin to develop their own lessons in the second unit of TOSS. Students read about Bloom's Taxonomy as it is applied by a mathematics teacher (Kastberg, 2003) and practice using this taxonomy as a tool for designing classroom questions, activities, and assignments that promote higher order thinking and connection-making. Next, instructors and readings introduce learning outcomes, and TOSS students practice writing outcomes for various grade levels. Students then identify various modes of instructions (e.g., lecture, discovery lessons, collaborative problem-solving sessions) and critically analyze each mode's benefits and challenges from both teacher and student perspectives.

After these discussions, TOSS instructors guide students through the development of a detailed lesson plan. Students begin to construct their own comprehensive lesson plans, both individually and in groups. Instructors require that each lesson plan submitted be aligned with GPS and include a strong opening activity and a closure. Instructors allow flexibility in delivery of new material so that TOSS students have the freedom to create student-centered discovery lessons that do not conform to the structure of traditional lesson plans. All group lesson plans are taught in class, with each group member participating in the delivery of the lesson. Classmates and instructors provide feedback. Students continue to hone their lesson planning skills when they create the unit plans that they will implement in their field experiences. The unit plan project requires detailed lesson plans for every day that students will teach in the field. A few weeks before the field experience, the students submit their unit plans to their instructors for feedback, and they then complete any required revisions. When unit plans are approved by TOSS instructors, students provide their plans to their collaborating teachers for their feedback and approval. Adaptation of lesson plans continues throughout the field experiences, and collaborating teachers and university supervisors coach students on the implementation of their units.

Upon completion of the unit on lesson planning, students learn how to assess the impact of their lessons on student learning. In the assessment unit, students read about and discuss strategies of formative and summative assessment. Bloom's Taxonomy is reintroduced in this unit as a tool for developing alternative assessments and open-ended questions. Students practice writing open-ended questions on various mathematical topics and receive feedback from their peers and instructors. Instructors demonstrate and discuss various questioning techniques, such as calling on individual students, wait time, and verbal feedback. Next, instructors begin lessons on test construction and scoring techniques. They provide students with sample mathematics tests, and students discuss strengths and weakness of each sample. In groups, students create their own tests and present them to the class for feedback. As a next step, each group creates a scoring rubric for the test that they created and present the rubric to the class. Rubrics are compared and contrasted, and instructors share examples of rubrics that were developed by other educators. The TOSS midterm is the culminating activity for this unit. The midterm assignment requires students to create a test on both mathematical and pedagogical material from TOSS and to design a rubric for scoring the test. On the day of the midterm, students exchange tests and complete the tests they are given. After test administration, the test creator grades the test, and each student writes a reflection on the test that he or she took with respect to the assessment techniques learned in the course. The midterm grade is based on the effectiveness and appropriateness of the test created, on its corresponding rubric, and on the reflection on the test taken (not on the performance on the test taken). Continued skill development in summative assessment occurs when students develop quizzes and the test for the unit plan that they create for the field experience. TOSS students also gain additional experience in formative assessment as they develop their questioning skills in the classroom, with feedback from their university supervisors and collaborating teachers.

After the lesson planning and assessment units have been completed, TOSS instructors turn their attention to classroom management. Students read about various classroom management strategies and policies, and they discuss Kohlberg’s stages of moral development, as interpreted by Thies-Sprinthall and Reiman (1990). Classroom discussions address appropriate classroom management strategies and discipline policies for various age levels. In groups, students develop sample classroom management policies and share them with their peers and instructors for feedback. Other activities include role play of discipline situations and shared experiences about appropriate and inappropriate discipline strategies that students have witnessed in the classroom (either as students or as observers). A second topic of this unit is parent communication strategies. Course instructors prepare various scenarios with parents that teachers might expect to encounter (e.g., confrontational parent/teacher conference, parent who wants advice on how to help her/his child be more successful, situations that might occur at open house). Groups discuss how to handle the various scenarios and then share their thoughts with the class. The culminating project for this unit requires students to write a letter to the parents of their future students. The letter should be appropriate to send home on the first day of school and should outline classroom rules and policies. Students have ample opportunities to practice and to adapt their classroom management and discipline skills, with feedback from their supervisor and collaborating teacher, while teaching their units during their field experiences. Some students even have the opportunity to observe parent conferences and other parent interactions with their collaborating teachers.

The final TOSS unit occurs when students return from the field experience. This unit begins with the creation of a comprehensive list of the types of diversity that students encountered during their field experience (e.g., ethnic, gender, family responsibilities, learning styles). Students then read and discuss various issues related to a diverse student population, such as the achievement gap and the need to vary instructional techniques to accommodate different learning styles. Discussions also challenge students to become aware of their own prejudices and biases that might influence their behavior and interactions in the classroom. As a culminating activity, students develop a set of strategies and considerations for successfully teaching a diverse population. Continued experience with the topics from this unit occurs during the student teaching experience.

Each of the five units in TOSS is designed to help students develop teaching skills and to provide meaningful opportunities to practice those skills, both in the classroom and in the field. Instructors are diligent in providing constructive feedback and in eliciting the feedback of classmates so that a variety of ideas and perspectives are presented. In addition, standards-based mathematics teaching methods are incorporated throughout the course, and topics and assignments are explicitly aligned with the Georgia Performance Standards. Throughout the semester, the students compile an online portfolio of their classwork and reflections that provides an excellent foundation from which they can build their student teaching portfolios.

Structure of the Field Experience

The four-week TOSS field experience is designed to provide students a first opportunity to implement their newly acquired mathematics skills in the public schools; however, students are provided significant structure, guidance, and support at all times. TOSS students spend two weeks in the field observing various classes and becoming acquainted with the various administrative and departmental structures in the school. They spend the other two weeks teaching the unit that they designed to two groups of students. This experience provides all of the conditions identified by Sprinthall and Thies-Sprinthall (1983) that promote teacher development. These five conditions are a significant new experience, guided reflection and analysis, balance between experience and reflection, support and challenge, and continuity.

The creation and implementation of the unit plan serve as the significant new experience. As part of the field experience, TOSS students complete assignments that require them to reflect on their teaching and observations. During the weeks that the students are not teaching, they complete ten formal observations. Five of these observations are focused on various aspects of the teaching of mathematics: content (i.e., the nature of messages that teachers relay about the characteristics and importance of mathematics), discourse (i.e., the types of questions that teachers ask), the teacher's homework practices, the teacher's use of class time, and teacher expectations and stereotyping (i.e., issues of equal access in mathematics teaching). The other five observations are the individual student's choice. They can observe other classes (mathematics or other disciplines), and they can observe other events in the school (e.g., assembly, ballgame, lunch period). For each of the ten observations, students are required to submit both a description of the observation and a reflection on their observations (i.e., their reactions and feelings about the classes and events they observed and their analyses of the aspects that were effective or that could be improved). In addition to the observations and reflections, students are required to submit overall reflections on their experiences in the schools and the effectiveness of their teaching.

Two major analytical and reflective assignments are required as part of the field experience. For the first assignment, students videotape a lesson that they teach, watch the video, and critique their own teaching by answering a series of questions about the various teaching skills they incorporated. The second major assignment is a program-required assessment, entitled Impact on Student Learning. To complete this assignment, students must gather data that enables them to determine the effectiveness of their teaching on an entire class, on two subgroups, and on two individuals. Students must also reflect on any patterns of student achievement that emerge through their analyses and must attempt to explain why such patterns might exist. To conclude the assignment, students must develop short-term and long-term strategies for addressing any identified weaknesses in teaching or assessment practices. Through these experiences and reflections, Sprinthall and Thies-Sprinthall's second and third conditions (guided reflection and analysis and balance between experience and reflection) are satisfied.

The university supervisors and collaborating teachers for the field experience provide

Sprinthall and Thies-Sprinthall's fourth condition, support and challenge. Instead of a letter grade, students receive a "satisfactory" or "unsatisfactory" mark for the field experience portion of the course. The non-graded nature of this experience allows the university supervisor and collaborating teacher to serve as mentors rather than evaluators. The collaborating teachers observe the students each day that they are teaching and provide constructive feedback and suggestions, as documented by a daily conference form. The university supervisor observes students at least once during their teaching experience and provides feedback and specific suggestions to challenge the student to improve.

Sprinthall and Thies-Sprinthall's final condition for teacher development, continuity, is satisfied by the design of the field experience. Rather than teaching a few days, students have an opportunity to implement and to assess an entire unit. Furthermore, during the next semester, they continue their teaching experience in student teaching.

The four-week TOSS field experience is designed to provide students with a highly structured first teaching experience that exposes them to some of the realities of teaching; however, they receive a great deal of support and feedback from both the collaborating teacher and the university supervisor. The students not only implement lesson plans of their own design, but they also analyze and reflect on their practice and have the opportunity to begin to address some of their weaknesses. Students consistently state that the field experience is the highlight of the TOSS course and that it was the most significant factor in preparing them for student teaching.

Benefits and Challenges of the TOSS Model

As with any instructional model, there are both benefits and challenges associated with the TOSS model. Perhaps the greatest benefit is that this model integrates theory, practice, analysis, and reflection. Students read and discuss theories that suggest skills that are likely to enhance student learning. Carefully designed classroom activities and course assignments allow students to practice these skills immediately. The field experience provides sustained practice, prior to student teaching that requires students to adapt and to generalize their skills to a real classroom setting.

The structure of the class meetings – three meetings a week for three-hour blocks – allows everyone in the group, instructors and students, to become well acquainted. Once a positive and professional dialogue is established, class discussions become very interesting and productive. The emphasis on group work also encourages collaboration and promotes the kind of teamwork that will be very helpful when TOSS students become full-time teachers. Classmates also become a support system, and excellent dialogue appears on the electronic class discussion board during the field experiences when students are not seeing one another everyday in class. The extensive time spent in class also enables instructors to establish strong mentoring relationships with students. If instructors are thoughtful, positive, and constructive in providing feedback, students begin to trust their instructors and are less defensive when instructors offer suggestions for improvement.

Another benefit of this course is the level of preparedness and motivation of the students. TOSS students are seniors who are very excited about their impending student teaching experiences. They have completed most of their mathematics and education classes and are ready to learn to apply their knowledge in the classroom. With very few exceptions, the students are an enjoyable group with which to work.

There are also some challenges inherent in the TOSS model from both instructor and student perspectives. Teaching the course requires a strong commitment of time and energy. Fortunately, at Kennesaw State University, teaching TOSS is an instructor's full teaching load for the semester. Planning for large class blocks is time consuming, as is the process of providing extensive feedback on the many course assignments.

In addition to planning and grading, coordinating the field placements and observations is time consuming and can be challenging as well. Because many of the students have to return to campus for afternoon classes, it can be challenging to secure placements with experienced teachers who have schedules that provide the opportunity to teach two sections of the same course during the morning hours. Driving to all of the schools to complete the observations during the four-week field experiences can also be a challenge, especially if some students are having a difficult time with teaching and need greater support and more visits from the university supervisor. The mathematics education faculty at Kennesaw State University, however, is committed to supporting the TOSS students; other professors are willing to assist with observations.

Another potential challenge that can arise in the field is a conflict between the student's and collaborating teacher's teaching philosophies. Sometimes, due to the pressure of high stakes testing and other factors, the collaborating teachers are not open to innovative teaching techniques and prefer that the TOSS student employ a more traditional teaching approach. In other cases, even with the collaborating teacher's support, students may be resistant to student-centered lessons because they are not accustomed to that type of instruction. When these situations arise, it is very important for the university supervisor to be available to the TOSS students in order to advise them on how to be successful in the teaching experience while not alienating the collaborating teacher or her students.

From the student's perspective, the TOSS course is also a great investment of time and energy, and students often become fatigued in the middle of the course when they are preparing and revising their unit plans in anticipation of the field experience. Most of them are taking other classes as well. To address this challenge, the TOSS instructors attempt to inform the TOSS students' other professors of the due date for the unit plans prior to the start of the semester. Some of these professors are willing to adjust the due dates of their major assignments in order to provide TOSS students time to complete this major assignment.

Some students find the transition from thinking like a mathematics student to thinking like a mathematics teacher very difficult. While some students can make this transition easily before the field experience, others find themselves forced very quickly to adopt a teacher perspective in the field. For some TOSS students, this transition can be overwhelming, and they are unable to complete the teaching requirement in their field experience. Those students have to repeat the field experience the following semester and, therefore, must delay student teaching.

Though the Kennesaw State University methods model has some inherent challenges, the benefits of the structure of the course outweigh those challenges. Students appreciate the opportunity to practice their teaching skills, both in the classroom and in the field, prior to student teaching. The relationships that the students develop through their collaboration in class often extend into their student teaching experiences and into their teaching careers. TOSS students continue to collaborate and to support each other long after the last class meeting. With strong support from administrators and from other department members, this course has been very successful in preparing student teachers and has become one of the most highly anticipated classes of the secondary mathematics education majors.

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