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Purposes and Goals of GAMTE

The purpose of GAMTE is to encourage and facilitate the improvement of mathematics teacher education across the state of Georgia. The goals of the organization are to: facilitate communication and collaboration among mathematics teacher educators between and within all educational levels; coordinate activities and work collaboratively with other associations, organizations, and governmental (national, state, and local) units to strengthen the mathematical, pedagogical, and clinical preparation of mathematics teachers at all levels (P-college); facilitate collaboration among mathematics teacher educators who are members of different academic units, such as departments of mathematics and departments of education; promote leadership among mathematics teacher educators in the broader mathematics education community; encourage research related to mathematics teacher education, especially which identifies factors that contribute to improving the preparation and professional development of mathematics teachers at all levels; encourage and organize programs and meetings focusing in issues related to the preparation and professional development of mathematics teachers; and foster the incorporation of appropriate technology into teacher education programs and professional development opportunities in mathematics at all levels (P - college).

A Letter from the President

The third annual conference of the Georgia Association of Mathematics Teacher Educators promises to be an excellent showcase of the collaborative work and ongoing scholarship of professionals from across our state. This year, our conference committee has assembled presentations by mathematics educators that represent ten different institutions and provide a wide range of original research projects and innovative pedagogical approaches for the preparation of mathematics teachers at all levels. The enclosed proceedings represent a small sample of the breadth of work of our membership over the last year.

This annual conference and proceedings is not the work of one solitary person. It is the result of many hours of work shared by a large collection of individuals. It is a testament to the fundamental strength of GAMTE: the collaborative nature of our members. Across the state, we are all facing the challenges of budgetary constraints, implementing and supporting a new K-12 curriculum, and responding to the professional development needs of our peers and students. This often results in all of us wearing multiple hats…as teachers, administrators, professional development instructors, graduate student mentors, curriculum developers, grant coordinators, and more. So it is often difficult to volunteer for one more project or to work on that external committee. But because so many of our members generously gave their time, we've been able to continue our work with GAMTE and have another opportunity to join together at Rock Eagle. Thank you to all who made this possible.

As we enter our fourth year as an organization, we will be fortunate to look to our newest leaders. Sharon Taylor, who has graciously organized our conference again this year, will step into the role of President-Elect with the rousing endorsement of our membership. She will be joined by Dianna Spence, as secretary, and Mary Garner as a Member-At-Large. We all look forward to their leadership and are grateful to the service Linda Crawford and Blanche Presley provided over the last two years.

Cindy Henning

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Abstract

The purpose of this study was to investigate the effectiveness of a mathematics professional development course. More specifically, in this study we examine whether geometric experiences have an impact on level of performance in mathematics. The van Hiele (Fuys, D., Geddes, D., & Tischler, R., 1988) model of geometric understanding provided a research framework from which to view geometric understanding. This model suggests five levels of understanding that should be taken into consideration when examining levels of geometric thinking: Visual, Descriptive/Analytic, Abstract/Relational, Formal Deduction/Proof, and Rigor.

The sample under study was three cohorts of practicing elementary teachers and mathematics coaches engaged in a 50-hour P-5 Mathematics Endorsement course entitled *Understanding Geometry*. Data collected through pre- and post-tests provided evidence that participants made significant improvement in geometric content knowledge and levels of understanding, thus verifying the effectiveness of their professional development experience. Also, this study points toward the importance of participants' entering level of understanding for achieving the course objectives.

Analysis of Achievement for Understanding Geometry

Given the commonly held belief that a causal relationship exists between teacher content knowledge and student achievement, it is essential that Georgia mathematics teacher educators contribute to strengthening the mathematics content knowledge of teachers. It is especially important to provide teachers with the knowledge and skills needed to effectively teach the new Georgia Performance Standards (GPS.) The GPS depend on building knowledge over time; therefore a strong foundation in P-5 is crucial.

The GPS promote a shift toward applying mathematical concepts and skills in the context of authentic problems. Students should understand concepts rather than merely follow a sequence of procedures. Like the National Council of Teachers of Mathematics (NCTM) standards, the GPS place a greater emphasis on problem solving, reasoning and proof, representation, communication, and connections. As students progress through the elementary years, learning opportunities that actively engage students through the use of manipulatives and various representations should be provided to allow them to grow in their geometric skills and understanding through grade-level appropriate activities such as:

- K – describe and sort objects
- 1st grade – observe, create, and decompose geometric shapes and solve simple problems including those involving spatial relationships
- 2nd grade – classify shapes and see relationships among them by recognizing their geometric attributes
- 3rd grade – broaden understanding of characteristics of previously studied geometric figures
- 4th grade – develop understanding of measuring angles with appropriate units and tools; understand the characteristics of geometric plane and solid figures
- 5th grade – expand understanding of computing area and volume of simple geometric figures; understand the meaning of congruent geometric shapes and the relationship of the circumference of a circle to its diameter.

Many recent professional development activities have been designed to improve teachers' knowledge of mathematics. In Georgia, it is possible to earn a P-5 Mathematics Endorsement by completing a sequence of four rigorous mathematics courses. One of these courses is *Understanding Geometry*. The Metropolitan Regional Educational Service Agency (MRESA) has received approval by the Professional Standards Commission to offer the Endorsement. To date, three cohorts have completed the sequence of courses under the same instructor and a fourth cohort is in progress. In this paper we will showcase the content and methods used to broaden understanding of fundamental concepts in geometry, construct and justify arguments, and interpret solutions, with a reference to the van Hiele theory of geometric understanding.

Statement of the Problem

Through this study, we sought to examine the effectiveness of the *Understanding Geometry* P-5 Mathematics Endorsement course on the performance of the participants. Specifically, we investigated the following question: *How do geometric experiences* *encountered in the* Understanding Geometry *course of the P-5 Mathematics Endorsement impact increased performances at higher van Hiele Levels?*

Professional Development Literature

Geometric Content Knowledge

Clearly, a critical component of mathematics teacher education is the acquisition of appropriate content knowledge. We agree with Tapan and Arslan's statement that, "the successful teaching of geometry at the elementary school depends critically on the subject knowledge of teachers." (2009, p. 1) In this context, however, it is important to clarify that the term "subject or content knowledge" means much more than the mastery of mathematical terms and procedures. Because "content knowledge" is a broad term with different levels of meaning, we would like to specify that we equate *geometric content knowledge* with *conceptual knowledge/understanding*. Although definitions of *conceptual knowledge* also differ, we will adopt the statement from Hiebert and Lefevre (1996, pp. 3-4) that conceptual knowledge is "knowledge that is rich in relationships… Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network." Educational experiences that include cooperative learning and reflective discussion enhance the construction of relationships, the depth of understanding and the likelihood of retention (Daniels et al., 1993; Garrity, 1998). Mikusa notes further that "[E]xploring geometry using manipulatives or computers, creating conjectures, and then arguing about those conjectures with classmates is essential in helping students develop the use of propositional knowledge with visual knowledge… [H]aving students try to convince others of their mathematical ideas not only forces them to reflect on their own ideas, but to elaborate these ideas, making them more mathematically explicit" (1995, p. 7).

Pedagogical Content Knowledge

Chamberlain and Powers (2007) note that the knowledge of mathematics for teaching is more than simply mathematics content knowledge. They add that it also includes a specialized knowledge regarding teaching, such as the ability to analyze students' mathematical thinking. This "specialized knowledge for teaching" is more commonly called *pedagogical content knowledge.* Reporting the results of her work with preservice and inservice elementary teachers, Fuller (1996) adopts Schulman's description of *pedagogical content knowledge*: "[PCK] includes… the most useful forms of representation of … ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations—in a word, the ways of representing and formulating the subject that makes it comprehensible to others… [It] also includes an understanding of what makes the learning of specific concepts easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning" (p. 9).

Battista and Clements (1995) report the results of their project in which 3rd- to 5th-grade students worked in pairs to determine the number of cubes in 3-D arrays. The results of this study convinced the researchers that reflection and cognitive conflict were essential components of learning. "[The students'] work illustrates that, like scientists, students are theory builders… The difference between the scientist and the student is that the student interacts with a teacher, who can guide his or her construction of knowledge" (pp. 8-9). Teacher education, whether for preservice or inservice teachers, must equip teachers to be the guides their students will need.

In her book, *The Middle Path in Math Instruction: Solutions for Improving Math Education* (2004), Shuhua An's perception of *pedagogical content knowledge,* as described by Jeremy Kilpatrick's review, includes a mathematics teacher's ability for "addressing and correcting students' misconceptions" (2005, p. 256). A very useful implication of An's findings is the importance of having teachers build their own conceptual understanding to enable them to identify and correct their students' misconceptions. In other words, for teachers to develop into the "guides" their students need, professional development courses must provide participants with frequent constructivist learning experiences.

Implications for Teacher Education

While a majority of the research studies related to teacher education involved preservice teachers, those involving inservice teachers or a combination of teachers used similar approaches. Those that we investigated focused on the importance of content and pedagogy. Olkun and Toluk (2004) described their success in a math methods course for preservice elementary teachers. The researchers focused on the development of both content and pedagogical content knowledge, moving their students toward more formal use of the concepts, as well as to a higher van Hiele level. Their method, which had three components: manipulatives, guided questioning, and collective argumentation, also resulted in an increase in concept retention. Fuller (1996), who worked with a combination of 26 preservice and 28 experienced elementary teachers, used a similar approach. She describes her method as "the synthesis or integration of teachers' subject matter knowledge into an understanding of how particular topics are organized, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction… [It] is that form of knowledge that makes teachers teachers…" Fuller further noted that research into the pedagogical content knowledge of both preservice and experienced teachers has shown that "teachers who themselves are tied to a procedural knowledge of mathematics are not equipped to represent mathematical ideas to students in ways that will connect their prior and current knowledge and the mathematics they are to learn, a critical dimension of *pedagogical content knowledge."* (p. 12, italics added)

A Geometric Understanding Theoretical Framework

A component of our research is the van Hiele theory regarding how students learn geometry. Van de Walle (2001) describes the van Hiele theory as "the most influential factor in the American geometry curriculum" (p. 309). Developed in the mid-to late-20th century by Dutch educators Pierre van Hiele and Dina van Hiele-Geldorf, the theory defines five levels (0 – 4) of geometric thought development. At Level 0 (Visualization), students think in terms of the shapes of objects and what they look like. They are able to group those that are "alike." Level 1 (Analysis) students are able to think in terms of classes of shapes rather than individual ones and to focus on properties. At Level 2 (Informal Deduction) students are able to use the relationships among the properties to classify shapes. Level 3 (Deduction) students are able to use the relationships among properties of shapes to formulate deductive proofs. Level 4 (Rigor) students have the ability to compare and contrast different axiomatic systems. The levels are sequential and movement from one to the next relies on geometric experiences, not maturation. (Fuys et al, 1998)

Olkun and Toluk (2004) state that they expect their pre-service elementary school teachers to be at van Hiele Level 3, given that they have all completed the secondary school geometry program. However, when Halat (2008) compared preservice elementary and secondary mathematics teachers, he found that the elementary group's average was below Level 2 and the secondary group's average was below Level 3. Attempting to teach geometry at a level which students have not reached does not work. So it is crucial that a diagnosis of van Hiele levels be done prior to and incorporated into the lesson planning.

Methodology

The Participants

The composition of the cohorts included in this study has varied due to increased communication of the expectations and refinement of the admission policy. The first cohort was comprised of classroom teachers from six different elementary schools in one of the 12 school systems in the MRESA service area. While twenty candidates were accepted into the P-5 Mathematics Endorsement, several self-selected out when they learned the rigorous demands of the courses. Of the fifteen who began *Understanding Numbers and Operations*, the first course, nine completed the requirements to earn the endorsement. Two candidates were not allowed to continue in the endorsement beyond the first course since they did not acceptably complete the unit requirement.

After the completion of the endorsement by the first cohort, instructors for each of the content courses, serving as the P-5 Mathematics Advisory Council, made some recommendations. The first recommendation was to provide an information session regarding the rigor and length of the endorsement. The second recommendation was that a placement test be given since the intent of the endorsement is to take good elementary math teachers and make them coach material. A placement test was created and administered to the other two cohorts included in this study. Individuals completing eight of twelve items correctly dealing with number and operations were admitted into the program. Those who completed fewer than eight items correctly were encouraged to take another course prior to re-applying for the admission to the Mathematics Endorsement. Retention of cohort members improved after implementation of these recommendations.

The second cohort consisted of teachers from a school cluster in another system at the request of their area superintendent. Seventeen of the twenty-two who took the placement test did well enough to be admitted. Ten of those seventeen began the endorsement, and seven completed it.

The third cohort consisted of nine individuals from different school clusters in the same system as the second cohort. The endorsement was open to any elementary teacher in the system. This cohort has completed three of the four content courses to date. Seven are progressing to the last course.

Means of Assessment

All participants from each cohort completed both a pre- and post-test. The test was compiled by two of the researchers using a collection of items from various sources. At the beginning of this study each item was categorized by van Hiele level. Some test items were not appropriate to classify at van Hiele levels; however two items were classified as Level 1 (Analysis - students are able to think in terms of classes of shapes rather than individual ones and to focus on properties) and five items were classified as Level 2 (Informal Deduction - students are able to use the relationships among the properties to classify shapes). One of the Level 2 test items examined participants' ability to classify geometric shapes by examining their properties – regular, irregular, and concave polygons for the first problem; and triangles, regular polygons and polygons with symmetry for the second problem. This item, Venn Diagram – Labels for Polygon Sort (see Figure 1), was a preliminary focus of this study.

> Polygons can be grouped in many different ways in addition to being grouped according to the number of sides. Two other ways included regular polygons and concave polygons. Regular polygons have sides that are all the same length and angles that are all the same size. Concave polygons look like they are collapsed or have one or more angles dented in. Any polygon that has an angle measuring more than 180 is concave. How should the Venn diagram be most specifically labeled?

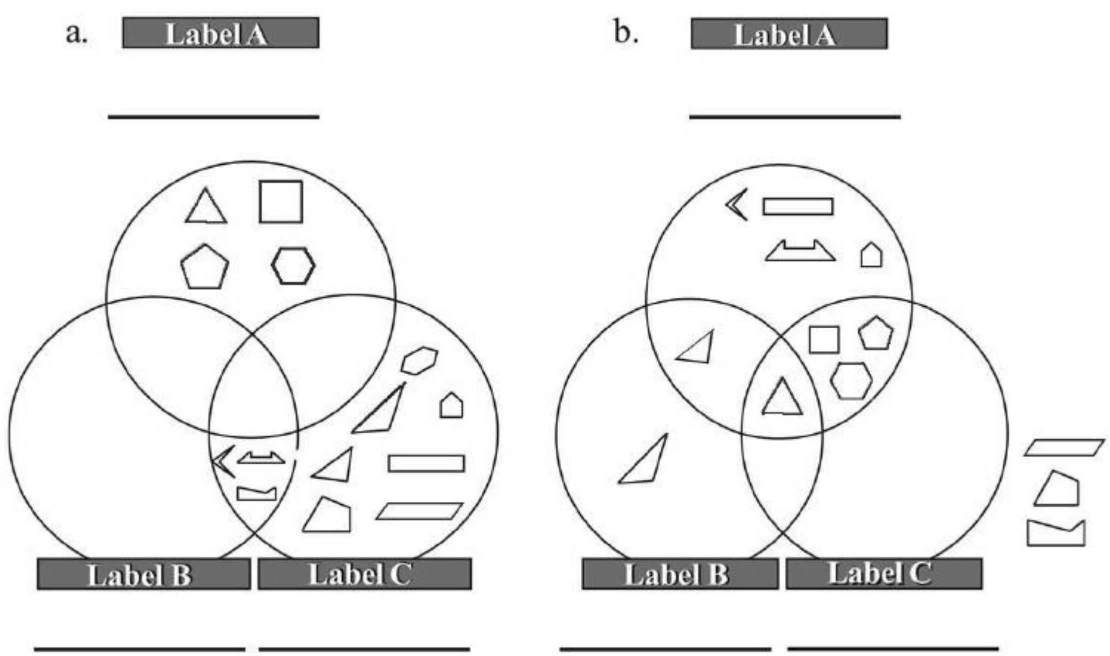

Figure 1: Venn Diagram – Labels for Polygon Sort

During the *Understanding Geometry* course, participants experienced a variety of activities sorting polygons such as Roping in Quadrilaterals (Gavin et al., 2001). In this activity participants placed sixteen given quadrilaterals into Venn diagrams they created from yarn. Once the quadrilaterals were sorted, appropriate labels were placed on the Venn diagram. Another activity was Mystery Rings (Gavin et al., 2001). In this activity participants progressed through six different tasks of increasing complexity, sorting quadrilaterals into Venn diagrams according to specified labels. For example, in Task 1 the labels are “At least one right angle” and “No right angles.” In Task 6 (using three rings) the labels are “At least two pairs of adjacent sides equal,” “All pairs of opposite angles equal,” and “All adjacent angles equal.” Participants also experienced an interactive online sorting activity, Sorting Polygons (www.learner.org).

Each of the three experiences occurred on Day 3 of the 9-day course. In each cohort group, participants expressed their value of and appreciation for these sorting activities. Typical comments were “This is such a rich activity” and “I love the way it gets harder and harder. I couldn’t do the last one if it had been first, but doing them in order I can get them all right.” Even though most participants found these activities challenging, they believed the experiences were appropriate for their grade 3-5 students.

Results

In examining the effectiveness of the *Understanding Geometry* course on the performance of the participants, we assessed increase in content knowledge and progress through the van Hiele levels. We began by analyzing aggregate differences in pre- and post-test scores by cohort for a particular problem. The results of this evaluation, plus consideration of changes in the entrance criteria for the course, led us to focus on Cohorts 2 and 3 as they were deemed more representative of future cohorts.

Specifically, aggregate data was compiled from all three cohorts and responses were examined on both parts of the pre/post-test item Venn Diagram – Labels for Polygon Sort. The data organization yielded four sets of data for each participant – problem A and problem B for both pre- and post-tests. Correct or incorrect responses on each Venn diagram label were compared from pre-test to post-test for each participant.

Each data set was comprised of three labels for the Venn diagram; therefore there were three possible correct answers. The analysis score was computed as post-test score minus pre-test score. If there was no change between pre- and post-test responses, a "0" was recorded. If one label (out of 3) showed improvement, "+1" was recorded. The best possible analysis score was +3 (all wrong on pre-test and all correct on post-test). If a participant scored better on the pre-test than the post-test that was recorded as -1, -2 or -3. Overall the possible analysis scores were -3, -2, -1, 0, 1, 2, or 3 for each participant on each problem. To compare cohorts, the scores were added for each problem, A and B. These aggregate scores are listed below:

Problem A, Cohort 1 (n=9) : aggregate score –3
Problem A, Cohort 2 (n=6) : aggregate score +3
Problem A, Cohort 3 (n=8) : aggregate score +4

Total across Cohorts (n=23) on Problem A : +4
Problem B, Cohort 1 (n=9) : aggregate score –4
Problem B, Cohort 2 (n=6) : aggregate score +4
Problem B, Cohort 3 (n=8) : aggregate score +6
Total across Cohorts (n=23) : aggregate score +6

Preliminary analysis of this data confirmed our perception that Cohort 1 was markedly different from Cohorts 2 and 3. The aggregate scores indicate that overall, participants in Cohort 1 scored better on the pre-test than on the post-test. In fact, only one participant from Cohort 1 showed an improved score on the post-test. Of the remaining eight participants, four showed no difference on pre- and post-tests and four showed a decline in accomplishment. Several factors could have impacted performance. Participants in Cohort 1 did not take the mathematics placement test required of subsequent Cohorts; nor did they have the benefit of an information session to advise them of the rigor demanded in the Endorsement courses. In addition, Cohort 1 experienced a more time-compressed course than either of the other Cohorts – nine class days in two weeks as opposed to nine class days in six weeks. The condensed pace, combined with unrealistic course expectations, may have created debilitating anxiety during the post-test. Alternatively, it is possible that the post-test was not taken seriously by this inaugural group of participants.

Changes in the course entrance criteria and course pace resulted in Cohort 1 being distinctly different from the other two cohorts, therefore we chose to focus our analysis on data gathered from Cohorts 2 and 3 which were deemed more representative of future cohorts. The analysis focused on changes in two outcomes: (1) content knowledge and (2) van Hiele level of understanding.

To determine changes in content knowledge, a paired t-test (null hypotheses $\mu_2 - \mu_1 = 0$ and alternative hypothesis $\mu_2 - \mu_1 > 0$) was conducted on pre- and post-test scores. The results showed a significant improvement ($p = 0.00001$) in participants' geometric content knowledge by the end of the course.

To determine whether progress had been made in van Hiele levels of understanding, a subset of pre- and post-test problems were identified as Level 1 (Analysis - students are able to think in terms of classes of shapes rather than individual ones and to focus on properties) or Level 2 (Informal Deduction - students are able to use the relationships among the properties to classify shapes) assessments based on the type of understanding necessary for correct responses to the items. Again, results of a paired t-test showed notable gains. There was a significant difference in the percentage of correct responses at both Level 1 ($p = .003$) and Level 2 ($p = 0.00000002$). It makes sense that the Level 1 growth would be smaller because many of the participants began the course with a Level 1 understanding. It is very gratifying to find such a large increase in the percentage of teachers who had achieved Level 2 understanding by the end of the course.

In addition, pre- and post-test scores for the van Hiele Level 2 problem <u>Venn Diagram – Labels for Polygon Sort</u> were compared. The analysis found participants' progress from pre- to post-test was also significant (A, p = 0.0143; B, p = 0.0093).

Future Plans

We will continue to examine future cohorts of the Mathematics Endorsement course entitled *Understanding Geometry* to confirm whether geometric experiences in that course have an impact on level of performance in mathematics. In addition, we will examine the progress of cohorts in one or more of the other three courses of the P-5 Mathematics Endorsement: *Understanding Numbers and Operations, Understanding Algebra, and Understanding Data Analysis and Probability*.

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Content Knowledge and Pedagogical Content Knowledge of Algebra Teachers and Changes in Both Types of Knowledge as a Result of Professional Development

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Abstract

In seeking to improve the mathematics education of all students, it is important to understand the connection between the content knowledge and pedagogical content knowledge of mathematics and how professionals can influence growth in both of these types of knowledge. We do not have an answer about the interplay of content knowledge and pedagogical content knowledge in successful instructional practices in the mathematics classroom. This study involves assessing the content knowledge and pedagogical content knowledge of secondary teachers of Algebra I. In addition, how are these types of knowledge expressed in instructional practices? Last, how do content knowledge, pedagogical content knowledge, and instructional practices change as a result of professional development which gives attention to increasing both types of knowledge?

Content Knowledge and Pedagogical Content Knowledge of Algebra Teachers and Changes in Both Types of Knowledge as a Result of Professional Development

Algebra I serves as a gateway course in dividing students into classes with significantly different opportunities to learn resulting in differences for future success in more advanced mathematics courses (RAND, 2003), to college preparation (Pascopella, 2000, Lawton, 1997, Chevigny, 1996, Silver, 1997, Olson, 1994), and for the preparation of the world of work (Silver, 1997). Teachers play a key role in ensuring that all students have the opportunities and experiences needed to learn mathematics (Mewborn, 2003).

What knowledge do Algebra I teachers need to posses in order to ensure all students have equitable opportunities to learn Algebra I? Limited research has been conducted in the areas of teacher content knowledge and pedagogical content knowledge of elementary teachers and these have shown that teacher's content knowledge is often thin and inadequate to provide the instructional opportunities needed for students to successfully learn mathematics (Ball, 1998a, 2003b, Ball & Bass, 2000; Fuller, 1996, Ma, 1999, Mewborn, 2001, Stacy, et al., 2001). Relatively few studies have focused on the content knowledge of secondary mathematics teachers, perhaps because of the belief that content knowledge may not be a problem at the secondary level because of secondary teachers' specialized knowledge of mathematics (Ball et al., 2001). However, the available research, while limited, has served to reveal the fallacy of this assumption (Ball et. al., 2001). At the secondary level, studies have considered teachers' content knowledge and pedagogical content knowledge in the areas of slope (Stump, 1997; Sherin, 2002) and functions (Even, 1993; Llinares, 2000; Sherin, 2002). Nathan & Koedinger (2000) studied teachers' perceptions about algebraic reasoning.

This paper relates to a study which focused on studying the content knowledge and pedagogical content knowledge of secondary teachers of Algebra I and how both types of knowledge changed as a result of professional development. In addition, changes in instructional practices resulting from changes in these types of knowledge were considered.

Content Knowledge and Pedagogical Content Knowledge

Teachers should be knowledgeable in the content areas for which they are responsible to teach. This must include a deep understanding of the mathematics they are teaching (NCTM, 2000), including both mathematical concepts and procedures (Conference Board of the Mathematical Sciences [CBMS], 2001). It will be difficult for any teacher to teach others about a subject if the teacher does not know the content himself/herself. The same is true for mathematics teachers. Mathematics teachers should know the mathematics they are teaching. But what exactly is it that they should know and how should they know it?

Content knowledge, however, is not sufficient and there is a difference between knowing mathematics and being able to teach it (Mewborn, 2001). Teachers also need knowledge of mathematics specifically used to facilitate the learning of mathematics by students (Sherin, 2002). Shulman (1987) first referred to this mixture of content knowledge and knowledge of pedagogy that belongs exclusively to teachers as "pedagogical content knowledge." Pedagogical content knowledge (Ball et al. 2003), bundles mathematics knowledge with the knowledge teachers have about learners, learning, and pedagogy (Ball et al. 2001). Pedagogical content knowledge can help teachers anticipate where students will have difficulties and be ready with alternative methods, explanations and representations related to a mathematical topic (Ball et al. 2001). In addition, pedagogical content knowledge includes representations that are most useful to teaching mathematics content (Ball et al. 2003).

Content Knowledge. For the purpose of this study, content knowledge will be defined as both the procedural and conceptual knowledge as well as the mathematical processes for using mathematics (Conference Board of the Mathematical Sciences [CBMS], 2001). This includes the teachers' ability to solve problems using a variety of methods, adapting to different contexts. In addition, content knowledge includes the ability to use reasoning and proof to make and investigate conjectures and evaluate mathematical arguments and be able to use algebraic reasoning in relationship to other mathematical topics. Teachers should have the ability to communicate mathematics so that others can learn and be able to listen to how others think about mathematics. Teachers should have the ability to make connections between mathematical topics, between the areas of mathematics, and to real-world problems. Teachers should be able to access different representations in organizing mathematics problems and should be able to translate between the mathematical representations and be able to model algebra in real world contexts. In addition, algebra teachers need an understanding of the use of technology in solving problems, and technology for exploring algebraic ideas and representations (NCTM, 2000; CBMS, 2001).

Pedagogical Content Knowledge. For the purpose of this study the following definition of pedagogical content knowledge will be used. Pedagogical content knowledge is viewed as the knowledge of a teacher to use his/her knowledge of mathematics to "unwrap" the mathematical topics and present the content in ways for students to successfully learn the mathematics (Ma, 1999). This knowledge includes the teacher's ability to use content knowledge to access different representations, as well as different methods for solving mathematics problems that may arise within the mathematics instruction (Ball et al. 2005). Pedagogical content knowledge also includes the ability of teachers to direct students to make connections between mathematical topics as well as helping them to see the connectedness of different representations for those same topics. Pedagogical content knowledge includes the ability of teachers to understand where and why students make errors and be prepared with alternative explanations and models (Sherin, 2002). Pedagogical content knowledge also includes the ability of teachers to respond productively to students' questions and pose problems and questions that are productive to student learning (Ball & Bass, 2003).

Methodology

This study consisted of two complementary investigations. First, survey research was conducted with a large pool of teachers in order to understand both their content knowledge and pedagogical content knowledge. Second, a multi-case study was used to provide an in-depth examination of these same types of knowledge, as well as to probe how teachers use their knowledge in mathematical instruction. The cases also offered opportunities to understand how content knowledge and pedagogical content knowledge changed as a result of professional development and to see how these changes are reflected in mathematics instructional practices.

Subject Selection. Multi-District Mathematics Systematic Improvement Program (MDMSIP) is a partnership between two universities and twelve school districts in East Alabama. The goal of MDMSIP is to improve mathematics education within the partnership districts. Teachers from nine secondary schools, who were accepted to be included in the summer professional development training as part of Cohort I of MDMSIP, along with an additional school that volunteered to be a part of the baseline data collection of MDMSIP, were used as subjects for the survey research.

Algebra Content Knowledge Instrument (ALCKIN). Because there was not an identifiable instrument to measure content knowledge and pedagogical content knowledge of secondary school teachers, one had to be developed. In order to develop test items for the survey research, consideration was given to the types of knowledge that teachers should possess in order to teach algebra to high school students along with the major algebraic topics teachers are expected to teach their students. Five key sources were consulted. The CBMS Report (2000) which includes recommended topic areas for the preparation of teachers was used. The RAND Report (2003) and Principles and Standards for School Mathematics (2000), both of which contain the big ideas in relationship to the types of algebraic knowledge in which students should be proficient, were consulted. Last, documents which provided specific objectives to be covered in the algebra courses at the secondary level were included. These two documents were the Alabama Course of Study and the Curriculum Guide developed through the NSF funded Multi-District Systematic Mathematics Improvement Project (MDSMIP). The keys areas of content knowledge focused on were families of functions, using algebraic structures in relationship to expressions, equations, and inequalities, analyzing change in various contexts, using algebraic reasoning in relationship to other mathematical fields, and properties of number systems.

Test development for the instrument began by considering the items from other assessments. The items in content instruments developed by the Learning Mathematics for Teaching (LMT) project (Hill, Schilling, & Ball, 2003) were multiple-choice in format and provided the opportunity for the researcher to see how teachers could solve problems that arise in the classroom. Additional understanding of teachers' mathematical knowledge would be gained by asking participants to explain why they selected particular answer choices. Thus, the quantitative summaries that were possible with the correctness of closed-ended items were merged with the deeper insights provided through the explanations in the open-ended explanations of these items.

A pool of thirty-five algebra items covering the identified content areas was developed for consideration to be used on the ALCKIN. Some of the mathematics problems were drawn from National Assessment of Educational Practices [NAEP]

(NCES, 2003b), RAND (2003), PSSM (NCTM, 2000), Stump (1997), Llinares (2000), and the LMT project (Hill, Schilling, & Ball, 2003). Additional items were developed to address areas of algebra content that were not covered by these items.

The initial pool of items was field tested to ensure the items were not confusing and to assure the instrument could be completed in a thirty-minute timeframe. The ALCKIN was field tested with seven students who were either undergraduate or doctoral students in mathematics education. All of the doctoral students had previously taught mathematics at the secondary level. Time was recorded as well as comments that were made about anything that was found confusing in the wording of problems. Successive revisions were made until the instrument was of an appropriate length, with well-designed tasks addressing the identified areas.

Sixty-five teachers involved with the MDMSIP gave consent to participate and were administered the ALCKIN. First, analysis of the results from the ALCKIN included correctness of their answers. Second, data from the written explanations was entered in *Atlas, ti* (Muhr, 1991) which is an analysis program used with qualitative research. Coding of the explanations included codes which were specific to particular test items. For example, the code "Passes vertical line test" may have only been used in Question one which contained graphs. Other codes may have related to more than one question. For example, "Drawings" may have been used as a reason for how students could show two algebraic expressions are equivalent or the participant may have actually used a drawing to clarify his/her explanation. After a code list was established, each document was revisited to ensure coding was appropriate. Each question was analyzed and conclusions were drawn for that particular question. Overall conclusions were then made for the instrument.

Figure 1 is question five from the ALCKIN. One area of consideration in the ALCKIN was using algebraic structures in relationship to expressions, equations, and inequalities. The emphasis for question five on the ALCKIN was not on testing the participants' ability to solve algebraic equations, but rather to give the participants an opportunity to find the solutions for an equation in a format that may be different from the usual method to which teachers and students may be accustomed. The participants were told that a teacher had asked his students to solve the quadratic equation $3x^2 = 4 - 2x$ using a spreadsheet. This particular problem was selected with the intended purpose that the resulting polynomial not be factorable and that the solution would involve a radical. A spreadsheet table was presented in which a range of values for x had been substituted in the expressions $3x^2$ and 4-2x. This table shows that the approximate solution for the quadratic equation should be between -1.5 and -1.6. Note that there is a second solution between 0.8 and 0.9, which is not shown in the table.

5. Mr. Casteel is using spreadsheets in his Algebra class to find solutions for quadratic equations. What approximate solution(s) for the equation $3x^2 = 4-2x$ should Mr. Casteel's students give using the following spreadsheet?

x	$3x^2$	4-2x
-1.8	9.72	7.6
-1.7	8.67	7.4
-1.6	7.68	7.2
-1.5	6.75	7
-1.4	5.88	6.8
-1.3	5.07	6.6
-1.2	4.32	6.4
-1.1	3.63	6.2
-1	3	6
-0.9	2.43	5.8
-0.8	1.92	5.6
-0.7	1.47	5.4
-0.6	1.08	5.2
-0.5	0.75	5
-0.4	0.48	4.8

Solution(s):

Explain your answer.

Figure 1. Question five from the Algebra Content Knowledge Instrument.

Cases. Selection of teachers for the cases was made on information obtained from the Alabama Department of Education website related to student achievement on the Alabama High School Graduation Examination [ASHGE] (ASDE, 2003). Initial classroom observations were made of eight teachers who all taught Algebra I, Algebra IA, or Algebra IB. Two of the teachers were selected from the highest achieving school, two from the lowest achieving school, and two teachers each from schools falling between the schools with the highest and lowest achieving students. However, the pool subsequently further contracted. At two of the schools, one of the two teachers observed was not going to be rehired for the upcoming academic year, meaning that continuing data collection would not be possible. At the third school, one of the two teachers did not plan to participate in the summer professional development, meaning that it was less likely that there would be any changes in his/her knowledge. Finally, at the highest achieving school, the teacher who was teaching Algebra I as well as Algebra IA was selected instead of the teacher who only taught Algebra I because variations in mathematics instruction would more likely be observed. Thus, there were four teachers in the final pool.

Data sources included the ALCKIN, two classroom observations of the four cases, teacher interviews of the four cases, field notes, researcher journal, and the Reformed Teaching Observation Protocol (RTOP). The Evaluation Planning Team of MDSMIP made the decision to use the Reformed Teaching Observation Protocol (RTOP) as part of the project's teacher observation process. The RTOP was developed by the Arizona Collaborative for Excellence in the Preparation of Teachers (ACEPT) at the Arizona State University (Sawade et al., 2000) to assess the degree to which the instructional practices of observed mathematics teachers are "reformed." The ALCKIN, classroom observations, teacher interviews, field notes, researcher journal, and RTOP were used prior to professional development to establish baselines for the content knowledge, pedagogical content knowledge, and instructional practices of the cases. With the exception of the ALCKIN, they were also administered after professional development to establish any changes that occurred in both types of knowledge as well as changes in instructional practices. Data sources from the cases were also entered into *Atlas, ti.* (Muhr, 1991). The codes developed in the survey study were used as a beginning point for coding the case study documents. New codes were added to this list of codes as the text from the documents, such as transcribed text from classroom observations and interviews, was analyzed. Additional codes were also added to capture events that may normally be observed in classroom observations but may not be a part of written instruments. For example, student conversation was a part of classroom observations and codes, such as “Student Gives Correct Answer” or “Student Indicates They Don’t Understand” had to be included. All of the documents were coded; the author then revisited each document to ensure consistency of coding throughout all of the documents.

Conclusion

The conclusion will first focus on the content knowledge and pedagogical content knowledge teachers had prior to professional development. Second, the type of instructional practices used by the cases will be examined. Last, changes in content knowledge and pedagogical content knowledge after professional development as well as changes in instructional practices will be addressed.

Content Knowledge. On parts of the ALCKIN that required a selection of answers, only seven of the twenty-five tasks were answered correctly by more than eighty percent of the participants. The ability to do mathematics procedurally was prevalent in explanations and in the procedures suggested in clarifying answer choices. The majority of the cases exhibited strong procedural knowledge and performed mathematical tasks using procedures.

Even though a majority of the cases exhibited strong procedural knowledge, procedural errors were made by participants on all items on the ALCKIN. Errors were made in the simplification of two algebraic expressions, arithmetic errors, errors in factorization, errors in substituting into the quadratic formula, and errors in writing inequalities.

The conceptual knowledge of all the participants in both studies was limited. Very few of the participants offered explanations or concrete examples to indicate they had a deep understanding of the mathematical task. Although teachers could use algebraic expressions, they had difficulty in explaining what an algebraic expression represented in an equation and responses from some of the participants suggest that as long as algebraic expressions are equivalent, it doesn’t matter how you write them. Some conceptual examples were used such as drawing pictures. While some participants did suggest the use of manipulatives, a conclusion cannot be drawn about whether these would be used specifically to develop mathematics conceptually or as another procedural method.

Gaps were exhibited in the understanding of topics on the ALCKIN, such as understanding of algebraic expressions, functions, and slope.

Participants in both studies had a difficult time in providing different representations for mathematical situations. Some of the participants did not recognize that algebraic expressions, although equivalent, cannot be used to describe different interpretations by students. Less than half of the participants could explain the meaning of an algebraic expression within the context of an algebraic equation. In addition, the majority of participants did not provide or use multiple methods for solving mathematical situations, and they tended to be quite procedural in nature. When the cases were pushed to provide other ways of solving mathematical tasks, even alternative procedural methods could not be provided.

Pedagogical Content Knowledge. The pedagogical content knowledge of teachers will be focused on the synthesized definition of this type of knowledge. The four areas of the definition will be addressed.

First, do teachers have the ability to "unwrap" mathematics topics and present them so students can be successful in learning mathematics? Actual mathematical instruction was not used in answering this question; however, if participants lack an understanding of mathematics topics it could be argued that they do not have the ability for unwrapping them and presenting them in ways so students can be successful.

Next, do teachers have the ability to access different representations as well as methods for solving mathematics problems that arise within mathematics instruction? Participants had a difficult time in solving the spreadsheet problem on the ALCKIN. The different representations of slope also caused a majority of the participants' problems in recognizing they were equally valid. The cases were able to use their pedagogical content knowledge in analyzing non-procedural methods used by hypothetical students within problem solving contexts during interviews.

Did the participants have the ability to recognize student errors and be able to respond to them with alternative models and explanations? Participants had difficulty in assessing student errors on the ALCKIN. Instead of responding to errors or statements made by students, participants generally ignored them or responded with how the student should have worked the problem using a set algebraic manipulation, rather than providing alternative explanations or models.

Did the participants have the ability to respond to questions, and to pose questions and problems that are productive to students learning mathematics? While the design of the ALCKIN did not directly address this issue, none of the cases exhibited this particular part of pedagogical content knowledge in interviews or in classroom observations.

Overall, it can be stated that teachers in this study had content knowledge. However, their knowledge was primarily procedural and they had limited conceptual knowledge. Also lacking was their ability to use various representations as well as use different methods to solve problems. We can claim that the participants did not have a deep understanding of the algebra content. Participants also exhibited limited pedagogical content knowledge. This was evident in all areas related to the definition of pedagogical content knowledge. Studies conducted with elementary teachers found teachers to have inadequate content knowledge and pedagogical content knowledge for successful mathematics instruction (Ball, 1998a, 2003b, Ball & Bass, 2000, Fuller, 1996, Ma, 1999, Mewborn, 2001, Stacy, et al., 2001). Similarly, results of this study indicate that algebra teachers also have inadequacies in these same types of knowledge needed for successful mathematics instruction to take place.

Content Knowledge and Pedagogical Content Knowledge in Classroom Instruction. Procedural knowledge was predominant within instructional practice. Cases demonstrated use of procedures within a variety of mathematics topics in almost all of the classroom observations. Three of the cases were not observed making procedural mistakes, while one case did. Conceptual knowledge of the cases, however, was seen to be very limited. There was not an occasion that could be identified where the cases exhibited that they knew why rules and algorithms worked. Cases were not observed using different methods of working problems. Conjectures were not investigated, nor was the cases' ability to develop and evaluate arguments observed either. Teachers did communicate ideas and clarify what they meant, but it was in superficial ways related to providing procedures and explaining what to do next. Cases were not observed making connections between mathematical topics. Representations of mathematics did not go beyond the procedures for doing mathematics.

When considering the pedagogical content knowledge, no unwrapping of mathematical topics or ideas was observed, only presentations of particular procedures. It was not obvious that teachers accessed different representations as well as different methods for solving mathematics problems. While teachers may have felt that their students were using alternative methods, in actuality they were generally the same procedures used by the teacher, but using a different order of steps or omitting some steps altogether. Since the teacher was dominant in mathematics instruction, little opportunity was given for other types of mathematical problems to arise that would necessitate different representations.

Connections between mathematical topics were not noted and since the cases did not use, or have students use, different representations for mathematical topics, this facet of pedagogical content knowledge was not present either. Analysis of student errors within the context of mathematical instruction generally related to incorrect mathematical computation errors, the teacher recognizing incorrect answers in algebraic expressions or equations, or using incorrect procedures in completing algebraic tasks. All student errors related to the mathematical topics covered during a particular day's instructions. Neither did results show teachers offering alternative explanations or models.

Cases could respond to questions their students asked during instruction, but the questions were generally about the correctness of an answer, if a student was doing the work correctly, or what the student should do next in solving the mathematical task. Moreover, teachers did ask their students questions productive in successful learning of mathematics, but they generally resorted to asking students to give short answers to arithmetic problems, the simplification of algebraic expressions, or to name the next step in a procedural process. Therefore, once again, the type of questions required of a teacher's pedagogical content knowledge was limited since these questions were not the type that would be productive to students learning mathematics in the ways they need to.

Changes in Knowledge and Instructional Practices. Changes in the procedural knowledge of the cases were not observed. However, changes in conceptual knowledge were evident and were expressed either by demonstration and/or in conversation. One of the cases demonstrated the use of algebra tiles in showing the addition of algebraic expressions while another case drew pictures of algebra tiles and a "magic square" in solving a quadratic problem. Both of these examples show that the cases could solve problems in different ways and could communicate their understanding clearly. They were also able to make connections to algebraic rules for factoring and it was obvious that they used different representations in demonstrating they understood how to solve the quadratic problem using different methods. All of the cases were able to suggest other non-procedural ways of solving the quadratic equation. Two of them offered conceptual ways of solving the problem.

Growth in pedagogical content knowledge of three of the cases was also evident. One case presented the use of algebra tiles in ways she said she used them in instructional practices. She provided the connections to using algebra tiles, which included using the tiles, drawing pictures, and bridging to the algorithms related to their use. Another case also demonstrated she was using algebra tiles for the multiplication of binomials and factorization. Although instruction was not observed using algebra tiles, students were observed making algebra tile drawings on the chalkboard during classroom observations.

Conversations with one of the cases included such terminology as "in-out" tables and how they could be used before introducing terms such as function, domain, and range. She also indicated that she had used them during instruction.

If classroom observations were the only venue used in considering how changes in content knowledge and pedagogical content knowledge were manifested in classroom instruction, the answer would be simple. Classroom observations of teachers' instructional practices before and after professional development showed very little difference in their content knowledge and pedagogical content knowledge. It can be noted that one of the cases was using an activity to help students develop the conceptual understanding for factoring the difference of two squares, but even in this one instance, she controlled the classroom conversation and limited the opportunities for her students to really develop the understanding they needed. However, when we consider the types of observations, such as the demonstration of algebra tiles with the two cases and the conversation about "in-out" tables, we might be able to suggest that it was possibly reflected in their instructional practice just not on the days of observation. Also, the display of student work from the Interactive Mathematics Program (IMP) units (Fendel et al., 2000) would suggest that changes in both of these types of knowledge are reflected in the instructional practices of the cases. These are limited views of pedagogical content knowledge, because they do not get everything the definitions of these types of knowledge entail. Furthermore, without these observations within the classroom context, we cannot say that it is used as a tool in helping students make sense of the mathematics or as just another procedure.

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Authentic Discovery Projects in Statistics

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Abstract

We report the activities and findings of a 3-year project, “Authentic, Career-Specific, Discovery Learning Projects in Introductory Statistics,” funded by the National Science Foundation. The project scope includes: 1) development of teaching materials for using discovery learning projects to teach statistics; 2) training secondary teachers to use the materials developed; 3) evaluation of student outcomes, in both content knowledge and attitudes toward statistics; and 4) extending and refining teacher training.

With input from an interdisciplinary team of instructors, materials were developed to assist the teacher in facilitating collaborative discovery projects using linear regression techniques and comparison techniques with appropriate t-tests. Web-based student and instructor guides authored to facilitate these projects are now available online.

Five pilot instructors used the materials in their classes. Data collected from the instructors included qualitative data about teacher experiences and observations while employing the materials, as well as quantitative data about student performance and attitudes. These findings were used to inform a one-day teacher training workshop on effective use of statistics projects with the materials developed.

Preliminary data analysis suggests that students in classes using the discovery projects achieve higher content knowledge and stronger perceived usefulness of statistics than do their traditional class counterparts.

Authentic Discovery Projects in Statistics

Introduction

The teaching of statistics has received increasing attention over the last two decades in mathematics education research. Researchers and educators have often suggested improvements to statistics teaching methods, especially those that focus on implementing the scientific method through authentic statistical experiences (Bryce, 2005). When best-practice pedagogies have been implemented in statistics courses, the results have been positive for achievement and for improved attitudes toward statistics. Research suggests that apprentice learning, wherein students complete real-world mathematics in authentic settings, develops better conceptual understanding and better knowledge transfer to non-mathematical and non-school settings (Boaler, 1998). Findings also suggest that statistics courses based on more constructivist models improve student attitudes toward statistics and that personal relevance is important for successful learning in statistics (Mvududu, 2003). One case study revealed that students learned more from a real-world project than from any other instructional component of a statistics course; the project also fostered an increase in student motivation (Yesilcay, 2000).

These findings in the literature prompted a curriculum development effort supported by a grant from the National Science Foundation. The grant, titled "*Authentic, Career-Specific Discovery Learning Projects in Introductory Statistics,*" has funded development of instructional materials, creation of instruments to measure the effectiveness of these materials, ongoing quantitative and qualitative research about the success of these teaching methods, and instructor training workshops to share materials and findings. The instructional materials were designed to facilitate student projects as a learning tool. We will describe these projects and their use in the classroom, the materials developed to support the projects, and the training of instructors to use these materials to facilitate the projects. We will then describe pilot instructors' experiences and observations as they implemented the teaching methods and materials that were developed. Finally, we will share preliminary findings regarding the impact of these teaching methods on student comprehension and attitudes.

Research Timeline

Although data collection and analysis are ongoing, most activities of the study are complete at the time of this writing. A second phase is planned (pending funding) to extend the teaching methods to a greater variety of settings and to train instructors more effectively. The timeline for the first phase of the study is shown in Table 1.

Summer 2007	• Development of instruments to measure student outcomes
Fall 2007 – Spring 2008	• Exploratory study on effect of these teaching methods • Development of interdisciplinary constructs • Development of web-based and printed teaching materials • Validation and revision of instruments • Selection of five pilot instructors • Control group data collection from all pilot instructors

Summer 2008	• Training of pilot instructors
Fall 2008 – Spring 2009	• Four pilot instructors complete experimental sections using methods and materials developed • Treatment group data collection
Spring 2009	• Training workshops conducted for statistics instructors
Fall 2009	• Experimental section for fifth pilot instructor
Ongoing	• Data analysis and dissemination

Table 1. Research Timeline

Curriculum Development and Student Projects

The curriculum development effort was directed by three goals:

1) Increase students' knowledge and comprehension of statistics
2) Increase students' perceptions of the usefulness of statistics
3) Increase students' self-beliefs about their ability to use statistics

The instructional materials that were developed assist the teacher in facilitating two collaborative discovery projects for use with elementary statistics content taught in a secondary or undergraduate class setting. The first project uses linear regression techniques, and the second uses comparison techniques with appropriate t-tests. Both projects simulate the real-world effort of scientists to hypothesize, collect and analyze data, and draw conclusions. Students select their own research topic, craft their own research questions, design surveys, collect their own data, apply the appropriate statistical methods to analyze the data, and report their findings in writing. Students also share their research with their peers in formal class presentations.

Interdisciplinary Research Constructs

As part of the curriculum development, an interdisciplinary team of instructors met to develop worthwhile research constructs for students. Team members wrote clear definitions of research constructs and provided instruments or instructions for measuring the construct quantitatively. Students were allowed to use these constructs and instruments if they chose; they were also allowed to create their own. However, the research constructs developed by the interdisciplinary team gave students a springboard of ideas from which to start, as well as a good set of examples of how constructs may be defined operationally, measured, and quantified.

Among the disciplines that were represented on the team were psychology, sociology, criminal justice, ecology, physical therapy, nursing, and education. In the field of psychology, suggested research constructs included perceived stress, perfectionism, anxiety, attention deficit disorder, and obsessive compulsive disorder. Accompanying each of these constructs was a brief screening instrument that could be administered and scored easily for purposes of quantitative research. In the field of sociology, suggested constructs included attitudes toward various social issues (e.g., corporal punishment, homosexuality, racism), all of which could be quantified on a Likert type scale. In the field of criminal justice, suggested research constructs included attitudes toward various criminal justice issues (e.g., gun control, death penalty, legalization of marijuana, pornography), all of which, again could be measured on a Likert scale. These examples are just a few of the research constructs that were assembled by the interdisciplinary team. The constructs proved interesting and engaging to students as they worked to select research topics for their own projects.

Materials Developed

Online student and instructor guides were developed to facilitate these projects. These materials are currently available online at http://radar.ngcsu.edu/~rsinn/nsf.

The student guide includes three sections: 1) an overall project guide, which describes each project and the steps needed to complete it; 2) a technology guide, which directs students to use the features of Microsoft Excel to implement the project; and 3) a variables and constructs guide, which includes research ideas assembled by the interdisciplinary team, as well as guidelines on creating a viable research construct and a corresponding data collection instrument (e.g., a well crafted, unbiased survey).

The instructor guide includes four components: 1) project overviews with suggested timeline, project implementation details, and best practices; 2) links to appropriate student materials that correspond with different phases of the project; 3) student assignment sheets; and 4) a variety of evaluation rubrics that can be used to score the projects.

Teacher Training

Five pilot instructors (one secondary and four college) were selected to test and refine the materials before they were more broadly disseminated. These instructors were given preliminary training through a series of meetings, one-on-one work sessions, and mentoring by the project researchers and authors of the materials.

After the pilot phase, a one-day workshop for statistics instructors, entitled the "Make It Real" Statistics Instruction Workshop, was hosted to disseminate the materials developed and to train teachers on effective use of these materials. All participants were high school teachers of Advanced Placement (AP) Statistics. Pilot instructors helped to deliver the training. Sessions conducted during the workshop included:

I – Designing Quality Variables and Constructs
II – Hands-on Survey Design Session
III – Project Organization, Phases, Assessment, and Rubrics
IV – Best Practices and Avoiding Pitfalls (Panel Discussion)
V – Technology Tools and Hands-On Data Analysis
VI – Team Presentations (Participants share their work product)
VII – Instructor Observations from First Implementations

Within 2 weeks of the workshop, a series of follow-up training sessions were conducted online to allow participants to discuss the material further, including details of their own plans for implementation. Instructors who so desired were eligible to earn one PLU (Professional Learning Unit) credit by participating in the workshop.

Instructor Experiences and Observations

As pilot instructors implemented these materials and teaching methods, they leveraged their own experiences to assemble a set of guidelines to help other instructors use such projects effectively. These guidelines were shared during the teacher training workshops. Some highlights of these guidelines address structuring and scaffolding the project, setting student expectations, and resolving potential issues with teams.

Structuring the Project

Projects are more successful when intermediate goals are set and students are required to submit defined deliverables along the way. For instance, students completing the regression project must first submit a list of several potential research ideas with specific variables, from which they will eventually select their final topic. After the topic is selected and approved, students must create a survey or other data collection instrument and submit this instrument to the instructor. The instructor often needs to help students refine the survey to set the stage for a more successful project. Once the survey is sufficiently refined and approved, students begin the data collection process, recording their data and beginning data analysis under some supervision. Sometimes an instructor will devote a day of class time for teams to engage in hands-on data analysis when the instructor is nearby for guidance. Finally, the team writes a report describing the research and the findings. This phase of the project is most successful if the students are given a template or outline of what should be included in the report. Some instructors also provide work samples from previous semesters for students to use as examples. By structuring the projects into these phases, instructors ensure that students stay on course and have a more successful experience overall.

Setting Student Expectations

Most instructors who have implemented these projects in the classroom will agree on the need to set student expectations early. Students often underestimate the effort and organization required to make their project a success. Many students also do not have a clear idea of exactly what they will do to analyze their data once the data have been collected; this part of the project is often a bit fuzzy in their minds (which is precisely why they need to do it for themselves.)

As one team of students elaborated on a post-project reflection:

> "The main thing that we have learned is that statistics take time. They cannot be conjured up by a few formulas in a few minutes. The time and effort that is put into a small research project such as this is significant. On a large scale, one can quickly understand the kind of commitment of money and time that is required just to obtain reasonable data."

Students will be much more successful if they know at the outset what to expect in terms of their own commitment of time and effort.

It is also a good idea to set student expectations regarding the findings of their data analyses. Students often expect to find strong correlations or highly significant statistical results, and upon obtaining some other outcome, they feel they must have done something wrong. In fact, researchers often obtain non-significant results; it is the appropriate interpretation of those results that is important. As another student noted:

> "While our results did not meet our initial expectations, this is not an utter disappointment. Before this project, statistics looked simple enough for anyone to sit down and do, but now it is evident that it requires more creativity and critical thinking than initially expected. Overall, it was an edifying experience."

If students are prepared for the possibility of getting results they might find disappointing, they seem more likely to believe the project was worthwhile.

Resolving Team Issues

Some students dislike team projects. Many instructors allowed students the option to complete a modified version of at least one of the two projects by themselves. Even among students who do not outwardly object to team projects, poor team dynamics can lead to a great deal of frustration. Some of this frustration can be avoided by establishing explicit guidelines for communication and cooperation among team members. Some instructors also define each team member's role and responsibilities to hold individual team members accountable for the final product.

Instruments

Three instruments were developed to measure the impact of these materials and teaching methods. These were a content knowledge test, a survey measuring the student's perceived usefulness of statistics, and a survey measuring the student's self-beliefs in their ability to use and understand statistics.

Content Knowledge Test

The content knowledge test initially contained 21 multiple choice items designed to measure student content knowledge in three areas: linear regression, t-test design and usage, and statistical inference in the context of t-tests. These areas corresponded to the strands of statistics directly addressed by the student projects.

During instrument validation, three of the content items were removed. This modification strengthened the reliability of the instrument and reduced data collection time for subsequent groups. The remaining items loaded onto the three factors that were intended for the content test (linear regression, t-test usage, and t-test inference). For the revised 18-item instrument, a reliability score of 0.67 was obtained through KR-20 item analysis for homogeneity. Given the three distinct groups of questions in the instrument, this score was deemed acceptable.

Perceived Usefulness Survey

The perceived usefulness survey was designed to measure students' perceptions of the usefulness of statistics, including the extent to which they expect to use statistics in their subsequent lives and careers. The survey contained 12 Likert-style items, each scored on a 6-point scale with descriptions ranging from "strongly disagree" to "strongly agree". Five of the items were reverse scored. Cronbach's alpha coefficient for this instrument was 0.93, suggesting a high degree of internal reliability. Because all items related to the perceived usefulness of statistics in general, no sub-scales were identified for the survey.

Student Self-Beliefs Survey

The student self-beliefs survey was designed to measure students' beliefs about their ability to use and understand statistics. Similar to the content knowledge test, this instrument contained three sub-scales; these sub-scales addressed self-beliefs with respect to concepts in: a) general statistics, b) linear regression, and c) the use of t-tests. The survey contained 15 Likert-style items, each scored on a 6-point scale. Cronbach's alpha coefficient for this instrument was 0.95, suggesting a high degree of internal reliability.

Preliminary Findings

This study was characterized by a relatively small expected effect size, combined with a high degree of variability among students. Because these factors resulted in tests with relatively low statistical power, the researchers found a significance level of $\alpha = .1$ to be appropriate. Findings are described for the initial exploratory study and for data collected during the pilot of the materials with the pilot instructors mentioned above.

Exploratory Study

The initial exploratory study was conducted in conjunction with the development of these materials, using the discovery project-based learning techniques promoted by these materials. The exploratory study consisted of 10 sections of elementary statistics at the same college, of which 6 sections comprised a control group (with traditional instruction) and the other 4 sections comprised an experimental group (using the discovery projects as learning tools). All sections were for the same course and were taught using the same text. There is also no reason to believe that the students registered for these ten sections differed in any way that would confound the results. At the end of the course, willing students in all sections completed the 2 surveys and the content knowledge test. From the control group, 164 students out of 192 (85%) participated in the assessment; from the experimental group, 113 students out of 128 (88%) participated. The results from t-tests conducted in the exploratory study are given in Table 2.

Instrument	Control		Treatment		*t*	*p*	Cohen's effect size *d*
	Mean	SD	Mean	SD			
Content Knowledge	8.8700	3.2400	10.8200	3.3650	4.825	.0000	0.59
Perceived Usefulness	4.2398	1.0113	4.5154	0.8570	2.442	.0076	0.30
Self-Beliefs	4.6997	0.8755	4.8164	0.6411	1.259	.1047	0.15

Table 2. Means, standard deviations, and t-test results from exploratory study

The treatment group outperformed the control group on the content knowledge test; the difference between the two groups was significant ($p < .0001$), suggesting that students in classes using the discovery projects had a stronger grasp of the content at the end of the course than did their counterparts in the control group classes.

On the perceived usefulness survey, participants in the treatment group scored significantly higher than did participants in the control group ($p < .01$), suggesting that students in classes using the discovery projects perceived statistics to have more utility than did students in the other classes.

Although the treatment group scored higher on average than the control group on the statistics self-beliefs survey, the difference between the two groups was not significant. A possible confounding factor is that students who have conducted statistics projects for themselves gain a new respect for the complexities of statistics that may not exist in classes where students do not conduct projects.

Pilot of Developed Materials

The pilot instructors first used the fully developed materials in Fall 2008. To account for learning curve effects, schedule differences, and discrepancies between secondary and college teaching schedules, some pilot instructors completed their treatment groups earlier than others, as shown in the timeline given previously in Table 1. Therefore, results are only reported here for instructors who had completed their pilot sections at the time of this writing.

In a quasi-experimental design, each pilot instructor collected data from his/her own sections of a traditionally taught course during the semester or academic year prior to implementing the materials (control group). Then each pilot instructor taught the course using the materials for at least two semesters (experimental group). Pilot instructors' control and experimental results were compared using the data collected from the three instruments as described. Of the five pilot instructors, one is still conducting pilot sections and collecting treatment group data. Results are reported only for the remaining four pilot instructors. However, the data were analyzed not only by each instrument, but also for each sub-scale of the content knowledge and self-belief instruments. This analysis helped to illuminate the impact of the projects.

Because data collection and analyses are ongoing, results are incomplete. At this stage of data analysis, results have only been examined by individual instructors; therefore, the samples are relatively small, restricting the power of the statistical comparisons. When all data are collected, whole group comparisons are expected to yield more significance. As the data illustrate, results also varied by instructor. Results for each instrument and sub-scale by instructor are given in Table 3.

The overall mean content knowledge score increased for each instructor, though in most cases, the observed increases were too small to be significant. Two instructors also saw a slight decline in content knowledge for t-test inference concepts (also nonsignificant). However, one instructor achieved significant gains in content knowledge for linear regression concepts, for concepts relating to the usage of t-tests, and for overall statistics content knowledge.

All four instructors saw a gain in overall student self-confidence in statistics, though the gain was significant for only one instructor. Average student self-confidence in general statistics concepts increased in the experimental group for all four instructors, and the difference was significant for three of them. Two instructors also achieved significant gains in student self-confidence for linear regression concepts. Although student self-confidence in t-test concepts increased for all four instructors, none of the gains were large enough to be significant. The perceived usefulness score was higher for three instructors' experimental groups than for their control groups, though the difference was significant for only one of the pilot instructors.

Scale	Instructor	Control			Treatment			*t*
		N	Mean	SD	N	Mean	SD	
Content Knowledge – Entire Instrument	**A**	41	7.17	2.365	24	8.50	3.093	1.817**
	B	20	8.45	2.762	28	8.54	2.502	0.110
	C	33	5.33	1.708	44	5.93	2.546	1.233
	D	43	6.86	2.178	76	7.05	3.253	0.385
Content Knowledge – Linear Regression	**A**	41	2.12	1.029	24	2.58	1.472	1.354*
	B	20	2.10	1.119	28	2.29	1.384	0.513
	C	33	1.24	1.032	44	1.50	0.928	1.132
	D	43	2.42	1.180	76	2.45	1.628	0.111
Content Knowledge –	**A**	41	3.54	1.398	24	4.33	1.949	1.756**
	B	2	4.50	1.638	2	4.93	1.464	0.934

Design/Usage of t-Tests		0			8			
	C	33	2.91	1.284	44	3.05	1.656	0.407
	D	43	2.58	1.314	76	2.88	1.657	1.087
Content Knowledge – Inference on t-Tests	**A**	41	1.51	0.925	24	1.58	1.018	0.281
	B	20	1.85	1.137	28	1.32	0.819	-1.776
	C	33	1.18	0.917	44	1.39	0.920	0.967
	D	43	1.86	0.915	76	1.72	0.974	-0.765
Perceived Usefulness – Entire Instrument	**A**	41	50.59	10.361	24	54.88	9.014	1.751**
	B	20	49.35	9.016	28	46.75	12.183	-0.850
	C	32	51.41	9.339	44	53.30	10.064	0.843
	D	43	50.23	10.963	76	50.92	11.722	0.321
Self-Beliefs – Entire Instrument	**A**	32	62.97	15.073	24	66.38	9.221	1.044
	B	20	55.95	20.289	28	63.71	12.304	1.523*
	C	33	63.21	9.512	44	65.25	9.022	0.951
	D	43	56.23	12.569	76	59.39	14.398	1.250
Self-Beliefs – General Statistics Concepts	**A**	32	17.94	4.272	24	19.33	3.116	1.414*
	B	20	16.45	6.057	28	18.64	3.540	1.452*
	C	33	17.39	3.316	44	18.95	2.853	2.168**
	D	43	16.26	4.042	76	16.80	4.505	0.680
Self-Beliefs – Linear Regression	**A**	32	21.50	5.292	24	22.71	3.520	1.024
	B	20	18.75	6.995	28	23.04	4.393	2.420**
	C	33	22.30	3.917	44	22.09	3.771	-0.239
	D	4	19.3	4.825	7	21.6	5.346	

		3	5		6	6		2.411**
Self-Beliefs – t-Test Concepts	**A**	32	23.53	6.091	24	24.33	3.964	0.596
	B	20	20.75	7.684	28	22.04	6.009	0.624
	C	33	23.52	3.598	44	24.20	3.638	0.828
	D	43	20.63	5.451	76	20.93	5.594	0.292

Table 3. Means, standard deviations, and t-test results for 4 pilot instructors
$*p < .10$, $**p < .05$

Summary

The method of using discovery projects to foster deeper understanding of statistics has shown promise. Students taught with this method have shown better content knowledge, greater self-confidence, and greater respect for the usefulness of statistics than have their counterparts in traditionally taught statistics courses.

As with most teaching techniques, the method in which discovery projects are implemented will have a direct impact on the success of the projects. Instructors find it helpful to structure the project with intermediate goals, to set student expectations early and clearly, and to establish clear guidelines for working effectively as a team.

Revisions and enhancements are planned for the instructional materials developed as part of this study. Instructor training workshops were well received by secondary classroom teachers. A second phase of this research project is planned, in which online teacher training modules will be developed and tested. The discovery-based statistics projects and the associated instructional materials and teaching methods should prove increasingly beneficial as teachers gain training and experience with them.

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