Proceedings of the Second Annual Meeting of the Georgia Association of Mathematics Teacher Educators

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Georgia Association of Mathematics Teacher Educators (GAMTE)

Purposes and Goals of GAMTE

The purpose of GAMTE is to encourage and facilitate the improvement of mathematics teacher education across the state of Georgia. The goals of the organization are to: facilitate communication and collaboration among mathematics teacher educators between and within all educational levels; coordinate activities and work collaboratively with other associations, organizations, and governmental (national, state, and local) units to strengthen the mathematical, pedagogical, and clinical preparation of mathematics teachers at all levels (P-college); facilitate collaboration among mathematics teacher educators who are members of different academic units, such as departments of mathematics and departments of education; promote leadership among mathematics teacher educators in the broader mathematics education community; encourage research related to mathematics teacher education, especially which identifies factors that contribute to improving the preparation and professional development of mathematics teachers at all levels; encourage and organize programs and meetings focusing in issues related to the preparation and professional development of mathematics teachers; and foster the incorporation of appropriate technology into teacher education programs and professional development opportunities in mathematics at all levels (P - college).

Letter from the President

Welcome to the second Annual Conference of the Georgia Association of Mathematics Teacher Educators! Isn't it nice to reconvene in this wonderful weather surrounded by the beautiful colors of fall at Rock Eagle? It's good to be here with all of you. I am revitalized whenever I spend time with Georgia's mathematics teacher educators. I am amazed all over again at what impressive people you are and at your tremendous commitment to mathematics education in Georgia.

We are fortunate to have a thought-provoking program again this year. Conference Coordinator Sharon Taylor and her committee reviewed the proposals and have put together a nice mix of research and practice-oriented sessions with presenters from seven different institutions. Presenters include experienced, well-respected researchers, as well as some researchers newer to the field. Thank you to Sharon Taylor and Susie Lanier of Georgia Southern for their hard work on this exciting conference program.

Examine the proceedings you hold in your hands. We may be the only state-level AMTE affiliate that publishes proceedings from our conference. Thank you to Proceedings Editor Lynn Hart of Georgia State and her committee (Joy Black, West Georgia College; Don Brown, Macon State College; Deborah Gober, Columbus State University; Linda Nash, Clayton State University; Marvin Smith, Kennesaw University; David Stinson, Georgia State University) for reviewing the papers and preparing those selected for publication.

Skip Fennell's April 2008 message in the *NCTM News Bulletin* discusses *validation* and makes the point that our work as teachers is validated by our long-term impact on our former students. I have recently received unexpected validation about the work we do because I have created a FaceBook page at the urging of one of my (yes, younger) colleagues in English. I happened on a few former students from Stone Mountain High School and many from Kennesaw State University. I have found (or been found by) more students as those networks have grown. Reconnecting with those former students has been unexpectedly rewarding and validating. Does any work other than teaching provide such an opportunity to impact the lives of young people?

Whether or not you receive the appreciation you deserve daily, your validation is all around us in the changes we have collaborated on in mathematics education in Georgia: rigorous, evidence-based teacher preparation; more mathematics understanding for teachers at all levels; and a rigorous, world class K-12 curriculum.

I am very grateful to have had the opportunity to serve you as GAMTE president. It has been an honor and a pleasure! You will be in good hands under the leadership of Cindy Henning, Blanche Pressley, and Linda Crawford, and our newly elected officers Nikita Patterson, Don Brown, and Debbie Gober.

Thank you for all you do for mathematics education!

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Becoming Critical Mathematics Pedagogues: A Journey

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Abstract

This session will report the findings of a study that explored the beginning transformations in the pedagogical philosophies and practices of three mathematics teachers (middle, high school, and 2-year college) who completed a graduate-level mathematics education course that focused on critical theory and teaching for social justice, and how these transformations are compatible (or not) with reform mathematics education as suggested by the National Council of Teachers of Mathematics (NCTM), and in turn, the new Georgia Performance Standards (GPS). The study employed Freirian participatory research methodology; in fact, the participants were not only co-researchers, but also co-authors of the study. Data collection included reflective essays, journals, and "storytelling"; data analysis was a combination of textual analysis and autoethnography. The findings report that the teachers believed that the course provided not only a new language but also a legitimization to transform their pedagogical philosophies and practices away from the "traditional" and toward a mathematics for social justice—a mathematics that is, indeed, compatible with the reform movements of the NCTM and GPS.

Becoming Critical Mathematics Pedagogues: A Journey

Since 1989 the National Council of Teachers of Mathematics (NCTM) has argued not only for instruction in mathematical *content standards*, but also for instruction in mathematical *process standards* (NCTM, 1989, 2000). The NCTM's, and in turn, the Georgia Performance Standards (GPS; see http://www.georgiastandards.org/math.aspx), recommendation of blending content and process standards throughout mathematics instruction requires the development of a different mathematics classroom—different from the "traditional" mathematics classroom found in most U.S. schools (see Hiebert, 2003, for a discussion of the traditional curricula and pedagogy in the mathematics classroom). In this different mathematics classroom, students are no longer passive, empty depositories awaiting the teacher's deposits—what Freire (1970/2000) coined, "the 'banking' concept of education" (p. 72)—but are active co-creators of classrooms "where students of varied backgrounds and abilities work with expert teachers, learning important mathematical ideas with understanding, in environments that are equitable, challenging, supportive, and technologically equipped for the twenty-first century" (NCTM, 2000, p. 4). These co-created classrooms, if desired, can set in motion a different mathematics pedagogy—a mathematics pedagogy positioned within critical theory.

Just as the NCTM (and GPS) recommends, mathematics pedagogy positioned within critical theory blends content and process standards throughout mathematics instruction in co-created classrooms; it differs, however, in that critical mathematics pedagogy centers instruction specifically around issues of social and political justice and reform (e.g., see Frankenstein, 1987, 1990, 1995; Gutstein, 2003, 2006, 2007; Gutstein & Peterson, 2005; Skovsmose, 1994, 2005). In other words, knowledge and understanding of mathematics from the perspective of critical pedagogy is understood as a means for student (and teacher) self-empowerment to organize and reorganize interpretations of social institutions and traditions, and to develop proposals for more just and equitable social and political reform (Skovsmose, 1994).

Although the concept of teaching for "social justice" is increasingly being emphasized in teacher education programs as part of teachers' overall "diversity" or "multicultural" initial preparation or professional development (McDonald, 2007), there is much less emphasis on how teaching for social justice—or critical pedagogy more generally—might be included in the preparation and development of mathematics teachers. For the most part, the mathematics education literature has reported the teaching practices of exemplar critical mathematics pedagogues (e.g., Marilyn Frankenstein, Eric Gutstein, and Ole Skovsmose). There is a scarcity of literature that has reported how mathematics teacher educators and/or programs might assist in developing critical mathematics teachers who teach for social justice (e.g., see Bartell, 2005).

Therefore, the purpose of this study (briefly summarized here) was to explore the transformations in the pedagogical philosophies and practices of three mathematics teachers (middle, high school, and 2-year college) who completed a graduate-level, critical mathematics education course that had the intended goal of assisting in the development of critical mathematics teachers. The study, however, was not about "teacher change" per se; we understand teacher change to be a complex endeavor (e.g., see Brown & Borko, 1992; Fennema & Nelson, 1997). In other words, we acknowledge effective mathematics teacher change most often occurs when teacher professional development opportunities are long-term, school-based efforts conducted within a community of learners that provide teachers opportunities to grapple with significant mathematics and to consider how students might engage with that mathematics (Mewborn, 2003). However, like the NCTM *Principles and Standards* (2000), we believe that teaching is a continual journey; in that, effective teachers do not "master" teaching, but rather find themselves in a continuous state of growth and change (Mewborn, 2003). Or, said in another way, effective teachers find themselves in a continuous state of becoming.

Building upon Bakhtin's concepts "ideological selves" and "ideological becoming," Gomez, Black, and Allen (2007) developed the idea of "'becoming' a teacher" (p. 2108). The concept *ideological self* acknowledges individuals as socially determined persons who view the world through a system of ideas that are mediated through language and experiences; whereas, the concept *ideological becoming* positions socially determined persons within continual conflicts and struggles as they come into contact with *different* ideological world views. Hence, *becoming a teacher*, as described by Gomez et al., is a process that is never finalized or fixed but rather a fluid process of continuous critical examination of self and students (and curriculum) in which old ways of thinking and acting are disrupted and transformed into new ways of thinking and acting—ways that hopefully are more ethical and just. It is within the context of becoming that the following three questions guided the study: (a) How did the teachers' beginning transformations change (or not) their philosophies of mathematics teaching and learning? (b) How did the teachers' beginning transformations change (or not) their classroom teaching practices? (c) How did the teachers' beginning transformations change (or not) their students' actions and learning?

Theoretical Framework

Critical theory provided the underlying theoretical framework for the study. The origin of critical theory is associated with the Frankfurt School (*circa* 1920), which holds a Marxist theoretical perspective: to critique and subvert domination in all its forms (Bottomore, 1991). Included in these critiques is an examination of how social interests, conflicts, and contradictions are expressed in thought and produced and reproduced in systems of domination (Bottomore, 1991). Critical theorists contend that an examination of these systems of domination will bring about an awakening of consciousness and awareness of social injustices, motivating self-empowerment and social transformation.

The concepts of self-empowerment and social transformation are reoccurring themes found in the scholarship of a contemporary critical theorist, Paulo Freire (e.g., see 1970/2000). Freire's literacy scholarship (but not limited to literacy) advocates a critical, dialectical reading of the word and world, so as to write the word to *rewrite* the world. It is his scholarship and his popularization of the concept of *conscientização*—"learning to perceive social, political, and economic contradictions, and to take action against the oppressive elements of reality" (Freire, 1970/2000, p. 35)—that provides, to a certain extent, the foundation for critical pedagogy. In the most general sense, critical pedagogy supports pedagogical theories and practices that encourage both teachers and students to develop an understanding of the interconnecting relationship among ideology, power, and culture and rejects any claim to universal foundations for truth and culture, as well as any claim to objectivity (Leistyna & Woodrum, 1996).

Critical pedagogy enacted in the mathematics classroom, by and large, adopts the pedagogical theories and practices of critical pedagogy, while explicitly using mathematics as an analytical tool for examining social injustices. Or more specifically, critical mathematics pedagogy is teaching mathematics for social justice. Gutstein (2006) noted that teaching mathematics for social justice has two dialectically related sets of pedagogical goals: one set focuses on social justice and the other set focuses on mathematics. Building from Freire's literacy scholarship, the social justice pedagogical goals are reading the world with mathematics, writing the world with mathematics, and developing positive cultural and social identities

(Gutstein, 2006). The mathematics pedagogical goals are reading the mathematical word, succeeding academically in the traditional sense, and changing students' (and teachers') orientation to mathematics (Gutstein, 2006).

Method

Grounded in critical inquiry (Crotty, 1998) and participatory action research (Kemmis & Wilkinson, 1998), the methodology employed in the study was Freirian empowering research (Lather, 1986). The goal of Freirian research is to blur the distinction between research, learning, and action by providing the researcher and the participants opportunities to collectively engage in the struggle toward social justice; it encourages researcher-participant reciprocity, turning participants into co-researchers (and in this case, co-authors as well) while providing the means for researcher and participants' *self*-empowerment (Lather, 1986).

The participants were selected from a group of 19 graduate students who completed a graduate-level course (spring 2006) entitled *Topics in School Mathematics Curriculum: Critical Theory and Teaching for Social Justice*. Three students were selected purposively as participants in order that a mathematics teacher representing middle, high school, and college would be included in the study. Data collection consisted of four artifacts written by the three participants: two written assignments from the course, reflective response essays written 3 months after the course, and autoethnographic "storytelling" narratives (Ellis & Bochner, 2000) written 9 months after the course. The artifacts from the course were reading journals and academic essays submitted by each participant. The third artifact, the response essay written 3 months after the course, asked each participant to respond to the three research questions of the study.

The fourth artifact, written 9 months after the course, was an autoethnographic narrative detailing each participant's experiences in planning and implementing a specific mathematics for social justice lesson within her respective classroom. Mary (middle school) and Carla (high school) used data about the "race" of people pulled over by police. Students used bar charts, box-and-whisker plots, and descriptive statistics to examine whether the data indicated racial profiling had taken place. Ginny (2-year college) used historical minimum-wage data, and students used regression to fit curves to the data and advocate for or against a raise in the minimum wage, and its amount.

Discussion of Findings

Throughout the collective autoethnographic narratives and textual data, the three teachers (i.e., participants, co-researchers, and -authors) articulated what it was like to attempt a new, different kind of mathematics teaching, one based in critical pedagogy. But this is a journey, not a destination. Mary, describing her critical pedagogical journey, declared: "Teachers must reflect on where they have been to see where they are going" (Academic Essay). Carla wrote, "I am trying to make the move toward a more democratic classroom where my students' voices are heard, their cultures have value, and everyone in the class is both a teacher and learner" (Response Essay). Ginny articulated her journey as developing a new way of life rather than a mere method of teaching: "Critical…[pedagogy] is about questioning everything, from the foundations of mathematics itself to every practice and belief. It is a way of life rather than a method of teaching. …I now find myself second-guessing everything I do, everything I plan, even my word choice in real time as I stand before a class" (Response Essay).

As each teacher spoke about her journey, each also noted that she had begun her teaching career with a more or less "traditional" belief structure about mathematics teaching and learning. Over the years, however, as experienced mathematics teachers, with several years of teaching experience, each had began to recognize that traditional practices were not working for every student—or for most students. Individually and collectively, they believed that their participation in the critical theory and teaching mathematics for social justice course provided them with a new language that assisted them in communicating and acting on what they were already thinking about mathematics teaching and learning. Carla wrote, “I realize now that 5 years ago I was already thinking like a critical theorist, I just didn’t have a clue what that was” (Response Essay). Ginny echoed and extended Carla’s remark, writing: “I began to grow on my own toward a more student-centered, equitable style, though I did not have the words for it or the feeling that what I was doing was being done elsewhere or would be respected by others” (Response Essay).

Effectively, this new language brought confirmation to what the teachers were, through their years of teaching experience, beginning to understand: “Unless educational methods are situated in the students’ cultural experiences, students will continue to show difficulty in mastering content area that is not only alien to their reality, but is often antagonistic toward their culture and lived experiences” (Bartolomé, 1996, p. 249). Or said in another way, “the only education that can have meaning is education that is personal and therefore political” (Lewis & Simon, 1996, p. 261). In many ways, the teachers’ narratives demonstrate the benefits of experiential learning, as advocated by Dewey (1938/1997). In each of the lessons, knowledge of subject matter—in this case, mathematics—was used to examine or make better sense of the *sociopolitical* lived experiences of the students (and teachers).

But it is not enough just to present problems based on something known to the students, given that the teachers understood that a fundamental tenet of critical pedagogy is the need to include both teachers’ and students’ voices and lived experiences in the learning process (Leistyna & Woodrum, 1996). The challenge for critical pedagogues therefore is how critical pedagogy might be employed to appropriate the more radical and useful aspects of contemporary cultural studies in addressing the different social, political, and economic contexts that are producing students, teachers, classrooms, schools, communities, and so on (Giroux, 1996). In other words, the active participation, interest, reflection, and critical understanding(s) of those taught—and those teaching—are necessary (Dewey, 1937/1987; Freire 1970/2000). The teachers believed that the most satisfying part of teaching mathematics from a critical pedagogical perspective was the conversations with and between their students. These conversations were not only culturally relevant, but personally relevant, even provocative; thus, achieving an essential aspect of social justice mathematics (Gutstein, 2006).

Each of the lessons planned and implemented contained both a personal and political element for students. During Mary’s (middle school) and Carla’s (high school) lessons in calculating basic statistical analyses of racial profiling data, students shared their personal experiences, both orally and in writing, unveiling the social injustices that occur in their communities—while *doing* mathematics (Stein, Smith, Henningsen, & Silver, 2000). For the adults in Ginny’s (2-year college) class, most of whom had had some experience working for minimum wage, the lesson planned to determine best-fit functions to minimum-wage data was very personal and led into political discussions as midterm elections were approaching. Many students were outraged upon realizing that the minimum wage had not changed in 10 years. In co-created classrooms like these, where content and process standards are continually integrated, the teachers argued, “once a fabric of relevance has been constructed, content learning naturally follows” (Kincheloe & Steinberg, 1996, p. 189). Or said in another way, when teachers create learning environments where students, especially those perceived as “low” performing, can demonstrate their possession of knowledge and expertise, they then demonstrate ability and competency (Bartolomé, 1996). In all three of the lessons, the teachers “tapped into” students’ knowledge that led to them taking personal ownership of the projects—and most important, of the mathematics.

In teaching for social justice, Freire (1970/2000) claimed, “The educator is the students’ partner as they engage together in critical thinking and a quest for mutual humanization” (p. 49). Though their students may not have been aware of it, the teachers were engaged in the process of learning as much as the students during the described lessons. The teachers noted that a traditional mathematics classroom typically generates little “new” knowledge for the teacher who is often accustomed to a banking method of teaching (Freire, 1970/2000), but in the lessons described, discussions revealed lived experiences and political opinions that presented both students and teachers with new knowledge. The objective during the socially just lessons was the production of students’ and teachers’ own ideas and values rather than the mere reproduction of ideas and values of the dominant groups (Leistyna & Woodrum, 1996), and, most important, using mathematics as a sociopolitical tool to justify and support these newly produced ideas and values (Skovsmose, 1994). Through this new, different way of teaching, students and teachers “develop their power to perceive critically *the way they exist* in the world *with which* and *in which* they find themselves” (Freire, 1970/2000, p. 83).

Teaching mathematics for social justice however asks much of teachers—and students—and it is not easy. Crotty (1998) claimed, and the teachers’ beginning transformations illustrate, that with every action taken, the context changes and one must critique her or his assumptions again and again. But the possible benefits of students and teachers engaging meaningfully with mathematics and transforming into agents of change are worth the work. The teachers believed that they, as well as their students, must “exercise the kind of courage needed to change the social order where necessary” (Giroux & McLaren, 1996, p. 318). The teachers acknowledged a choice between a pedagogy that accepts the status quo and a pedagogy that seeks to bring about change (Crotty, 1998)—they were committed to choosing the latter.

Since completing the course, and teaching the social justice lessons described, the teachers claimed that they continue to evolve as critical mathematics pedagogues. They actively seek and encourage critical connections with other disciplines. They continue to use the tenets of critical pedagogy in planning curricula, developing classroom environments, and establishing channels of communications. In other words, they have become stronger facilitators of critical mathematical discourse not only with their students, but also with their colleagues.

On the other hand, the teachers also conceded that their pedagogical philosophies had changed faster than their practices—not an uncommon phenomenon among mathematics teachers (e.g., see Wilson & Goldenberg, 1998). While they all agreed that their classes should be more a conversation between teacher and students (Lerman, 2000) and move away from the “banking” method, they found themselves mired in traditional practices. They noted many constraints that often make it difficult to change their pedagogical practices:

> At first I am always met with some student resistance as I encourage my students to become independent thinkers as they work together to discover patterns and formulate conjectures. (Mary’s Academic Essay)
>
> My biggest challenge this semester, however, has not been teaching but convincing my colleagues of what to teach and what not to teach. (Carla’s Academic Essay)
>
> I am limited to single semester, single subject, strictly defined curricula. I am not easily able to do some of the long-term projects. (Ginny’s Academic Essay)

Furthermore, Mary and Carla, as Pre-K–12 educators, experienced difficulties implementing social justice lessons into their classrooms while at the same time undergoing state curriculum reform (i.e., GPS). Although each teacher discussed a specific constraint, each felt she had experienced somewhat similar constraints that prevented her from making the change more quickly. But then again, all three teachers also asserted that they would be diligent in developing methods that overcome or undercut these constraints as they continue to establish the tenets of critical pedagogy as integral components of their pedagogical philosophies and practices.

Conclusions

The study began with three questions in mind: how did exposure to critical pedagogy begin a transformation (or not) of the teachers' philosophies of mathematics teaching and learning, their classroom teaching practices, and their students' learning. With regard to their philosophies, it is not so much that their philosophies changed, but that they now have a name for what they were already thinking. And once it was named, their way of thinking gained legitimacy in their eyes, as well as in others' view. The teachers believed that they could stand up proudly and proclaim themselves, through words and actions, as critical mathematics pedagogues, rather than feeling insecure and keeping quiet.

The teachers' mathematics classroom teaching practices have definitely changed. Most important, the teachers' have been given a new "voice." Each teacher felt as though she had already begun a journey toward reform mathematics instruction, but again, critical pedagogy gave her the words and the backing to put her ideas into action. Collectively, the teachers provided specific examples of lesson plans created and implemented using critical pedagogy, but mostly the teachers believed that there have been innumerable instances when their new *critical* outlook has influenced their choice of words, their decision to allow a discussion to stray from mathematics into something equally meaningful, and a generalized awareness of what was going on in their classroom from more than just a mathematical perspective. The teachers said that their classrooms have become more democratic, inclusive places, with class sessions moving toward a conversation between teacher and students. And perhaps most important, the teachers believed that they have an ongoing sense of constant change and improvement, very different from the traditional idea of there being a "best practice" that a teacher should learn and use forever. In short, each has a sense of becoming.

Their students' learning also has been positively impacted; in that, the students too were given a new voice through the use of critical pedagogy. The students became more engaged with the mathematics and stayed on task during the mathematics projects. Students that normally did not contribute were motivated to participate in classroom discussions. Communication outside of the classroom was opened as well; they talked with their parents about the lesson, they came to office hours, and they attended after-school tutoring sessions. These actions might have been because they found the mathematical topics personally relevant and were able to connect it to their studies in other classes.

For those who believe in the transformative power of critical pedagogy, the significance of the study's conclusions cannot be overstated. There are probably many mathematics teachers out there, like the teachers of this study, afraid to pursue the ideas they have because they lack the language and the legitimization; forced, through lack of knowledge, to reproduce the same traditional mathematics pedagogy they themselves endured. If those teachers could be reached, imagine how many students they could influence. Dewey stated, "If a sufficient number of educators devote themselves to striving courageously and with full sincerity to devote themselves to find the answers to the concrete questions which the idea and the aim put to us, I believe that the question [of education and social change] will cease to be a question, and will become a moving answer in action" (Dewey, 1937/1987, p. 417). Of course, a teacher cannot be said to have chosen a method or philosophy of teaching if only one option is offered. Because the teachers now consider themselves *becoming* critical mathematics pedagogues, they hope that teacher education programs will expose other preservice and inservice teachers to different options, such as critical pedagogy, to move mathematics pedagogy away from the traditional—and toward a pedagogy of social justice.

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Virtual vs. Concrete Manipulatives in Mathematics Teacher Education: A Call for Research

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Abstract

Are virtual manipulatives as effective as concrete (hands-on) manipulatives to build conceptual understanding of number concepts and relationships in pre-service middle grades teachers? In the past, the use of concrete manipulatives in mathematics courses for Clayton State University's preservice middle grades teachers has proven to be a very effective way to build conceptual understanding of a variety of mathematical topics. This paper presents an argument for the need for research into the usefulness of virtual manipulatives for enhancing mathematics teacher education and their potential to supplement (or replace?) concrete manipulatives.

Virtual vs. Concrete Manipulatives in Mathematics Teacher Education: A Call for Research

Introduction

Since the inception of Clayton State University's (CSU) Middle Grades Teacher Education program in 1993, hands-on manipulatives have been used very successfully in the junior-level Number Concepts and Relationships (MATH 3010) course. In addition to a textbook, students were required to purchase a manipulative kit that included, among other things, Cuisenaire Rods, pattern blocks, fraction circles, and two-color counters. Students worked collaboratively to build conceptual understanding of the arithmetic of fractions and integers, using those manipulatives. That understanding was assessed by small group and whole class discussions and the use of performance tests.

Certain results occurred consistently, year after year, class after class. Invariably, students declared, "I wish I had been taught this way. This makes so much more sense." And "I always hated *(fill in different topics—fractions, negative integers, etc.)*, but now I understand it." Some even admitted liking math for the first time! Those statements were usually made by the "less-mathematically-inclined" students. In fact, those students were consistently more adept at learning to use the manipulatives than the traditionally higher achievers; it was much harder for the latter to let go of the rules they had learned and to operate on the concrete level required for successful use of the manipulatives.

Because the junior-level mathematics courses in CSU's teacher education program combine content and pedagogy, students studied the NCTM *Standards* and reflected, both orally and in writing, on the importance of providing appropriate experiences and instructional materials to facilitate students' mathematics learning. Communication and reflection were emphasized throughout the course, contributing to the overall success of using manipulatives to build conceptual mathematics understanding.

The Problem

With the advent of the Georgia Performance Standards (GPS) reformation of the pre-K—12 curriculum, as well as the prevalence of websites providing free access to virtual manipulatives, the time is right for Georgia mathematics teacher educators to investigate the advantages and disadvantages of supplementing (or replacing?) concrete manipulatives with virtual ones. The GPS curriculum advocates the constructivist approach to mathematics learning and provides tasks to help teachers implement it. (http://www.georgiastandards.org/math.aspx) In addition, the new curriculum incorporates the TIMSS recommendation to teach fewer topics in more depth, thus allowing more time for the incorporation of manipulatives. (http://nces.ed.gov/timss/results.asp)

The ubiquitous use of various forms of technology by today's youth provides additional incentive to investigate the usefulness of virtual manipulatives. Might they be more motivating to students, especially those in middle and high school, than the more toy-like concrete manipulatives? On the other hand, should school funding be directed toward the more expensive technological resources, especially given the current economic slump? Considering the meaning of "conceptual understanding" and the often omitted component of bridging from the concrete to the abstract, is there a difference between concrete and virtual manipulatives in the ease of that transition? It is important for groups such as GAMTE to investigate these questions.

The teacher education classroom provides the perfect setting in which to examine the effectiveness of hands-on versus virtual manipulatives, while providing experiences in which pre-service teachers develop a deeper understanding not only of the mathematics but also of the resources themselves and their respective pedagogical impact. In other words, teacher educators have the opportunity to do research while facilitating not only the learning of mathematical concepts but also of pedagogy and pedagogical research methods.

Current Research

Conceptual Understanding

Because we are concerned with the enhancement of *conceptual understanding*, it is imperative that we define the term. Although definitions differ, we will adopt the statement from Hiebert and Lefevre (1996, pp. 3-4) as cited by Star (2005, p. 406), that conceptual knowledge is "knowledge that is rich in relationships… Relationships pervade the individual facts and propositions so that all pieces of information are linked to some network." The relationships formed by the use of manipulatives incorporate visual, tactile, and kinesthetic experiences. Adding cooperative learning and reflective discussion further enhances the depth of understanding and the likelihood of retention. (Daniels et al., 1993; Garrity, 1998.)

Pedagogical Content Knowledge

Broadening our net to include a focus on pre-service teachers' construction of conceptual understanding in a course that incorporates content and pedagogy, it is also important to understand the term "pedagogical content knowledge." In his review of Shuhua An's book, *The Middle Path in Math Instruction: Solutions for Improving Math Education* (2004), Jeremy Kilpatrick notes An's perception of pedagogical content knowledge as including a mathematics teacher's ability for "addressing and correcting students' misconceptions" (2005, p. 256). Kilpatrick further notes An's observation of the teachers in her study having limited success "bridging from manipulative materials to mathematical ideas" (p. 258). For me, the useful implications of An's findings are (1) the importance for teachers to build their own conceptual understanding to enable them to identify and correct their students' misconceptions, and (2) the need to incorporate "bridging construction" into the course.

Research Involving Concrete Manipulatives

Regarding the pedagogical impact of *concrete manipulatives*, there is a plethora of available information. Based on their own experiences, authors of the NCTM S*tandards* from 1989 and 2000 recommended giving students "experiences in using a wide range of visual representations" (e.g., *concrete manipulatives)* to solve mathematics problems (2000, p. 284). Many researchers (e.g., Van de Walle, 1973; Grouwns, 1992; Vinson et al., 1997) have found a connection between the use of manipulatives and a decrease in students' math anxiety levels. In their study of pre-service mathematics teachers, Vinson et al. reported that the use of manipulatives served a "two-fold" purpose:

> "First, the concrete experiences aided in preservice teachers having a better understanding of the mathematical concepts and purposes for procedures. Secondly, the use of manipulatives assisted the preservice teachers in learning how to teach with more than just modeling a procedure on the chalkboard…" (p. 8).

Research has also shown the use of manipulatives in the mathematics classroom to be motivating. In her study of high geometry students, Garrity (1998) found that using manipulatives and cooperative groups motivated her students. She concluded

> "In order to give meaning to math teaching, students are best served by learning concepts by actual manipulation of physical materials. Motivation is best accomplished when there is an active involvement with physical objects" (p. 21).

Garrity cites other researchers who also support the "manipulation of "physical objects" as a deterrent to "pseudo-learning" (Carin & Sund, 1975, p. 338) and who recommend the inclusion of "opportunities for reflection" to balance effective learning practices (Daniels et al., 1993, p. 9). In a study using concrete manipulatives in two 8^{th}-grade pre-algebra classes, Hinzman (1997) reports that students' mathematics performance was enhanced and their attitudes were significantly more positive than those of students from previous years. (These results were well founded despite the fact that Hinzman's data analysis appears to be flawed.)

Research Involving Virtual Manipulatives

Because the advent of *virtual manipulatives* is relatively recent, the research regarding them is less prevalent than that of concrete manipulatives. Moyer et al. (2002) define a *virtual manipulative* as "an interactive, Web-based, visual representation of a dynamic object that provides opportunities for constructing mathematical knowledge."(p. 185) In addition, they identify one's interaction with virtual manipulatives as an example of the process of representing mathematics recommended by the NCTM *Standards*.

> "Because it is advantageous for students to internalize their own representations of mathematics concepts, interacting with a dynamic tool during mathematics experiences may be much more powerful for internalizing those abstractions." (p. 5)

Noting that "students learn in different ways," Schackow (2006-2007) describes several activities for which mathematics teachers can use virtual manipulatives to teach fraction concepts to middle school students. She recommends the National Library of Virtual Manipulatives website (http://nlvm.usu.edu/en/nav/vlibrary.html) as one that contains many worthwhile activities. Schackow, however, has expanded the list of concepts for which certain NLVM manipulatives can be used. For example, she illustrates how to use NLVM's virtual Color Chips, Geoboards, and Pattern Blocks to model computations with fractions. In addition, she lists several advantages of virtual manipulatives. They are

- Available—"Teachers may be limited in the quantities and types of concrete manipulatives available to them." (p. 10)

- Time-saving—"Teachers may not have time to make their own manipulatives." (p. 10)

- Motivating—"(Middle school) students may find working on a computer with virtual manipulatives more desirable than using concrete manipulatives that they might view as childish." (p. 10)

Schackow concludes,

> “Using the virtual manipulative activities discussed in this article can help students deepen their conceptual understanding of fraction computations and avoid such struggles and frustrations…(and) may lead to student exploration and classroom discussion that will enable students to make sense of fraction computations.” (p. 10)

Several studies compared the use of concrete and virtual manipulatives to teach mathematics. Brown (2007) conducted an experiment to determine if students who used virtual manipulatives would out-perform students who used concrete manipulatives. The subjects in her study were 48 6th-grade students in an urban public school. Her results indicated that students who received instruction with concrete manipulatives out performed students who used virtual manipulatives, but that both types of manipulatives enhanced the learning environment. Brown’s results are suspect, however, since there were differences in academic ability of the two groups. In addition, the types of virtual manipulatives used (fraction bars) were different from the types of hands-on manipulatives used (pattern blocks). Both differences could have influenced the results of the study. It would be interesting to determine whether the results would have been different if the lower-ability group had used the concrete manipulatives and the higher-ability group the virtual.

In her report, Brown cites other studies that investigated virtual manipulatives. Enderson’s study (1997) showed that using virtual manipulatives to explore volume of a box expanded students views of the mathematics involved (p. 10). Reimer and Moyer (2005) reported increased success teaching fractions with virtual manipulatives over paper-and-pencil instruction. Brown quotes Reimer and Moyer as indicating that virtual manipulatives’ ability to “connect dynamic images with abstract symbols” (pp. 10-11) is an advantage over concrete manipulatives. Brown (2007) adds the advantage that virtual manipulatives take less time to manipulate than concrete ones. Studies by Olkun (2003) and Dorward & Heal (1999) indicate that virtual manipulatives are as engaging and provide equally as strong an effect on mathematical understanding as do concrete manipulatives.

Implications for Teacher Education

A survey conducted in Australia by Howard et al. (1997) to determine the use of manipulatives among primary and secondary mathematics teachers raised questions regarding the issue of whether teachers’ acceptance of the usefulness of manipulatives has “a solid conceptual base” (p.9). The researchers also indicated that

> “There is a clearly expressed need…for further training in the use of manipulatives in mathematics teaching”, a fact that “has implications for both pre-service teacher education programs and teacher development sessions” (p. 9).

Surely the same need exists in the United States. Certainly, the NCATE/NCTM Program Standards (2003) for Middle Level Mathematics Teachers and the GPS Process Standards also support investigation of the role of virtual manipulatives in the teaching and learning of mathematics.

Perhaps the most compelling charge regarding the role of virtual manipulatives in mathematics education comes from two Turkish educators, Durmus & Karakirik (2006). They define a *concrete experience in a mathematics context*

> “not by its physical or real-world characteristics but rather by how meaningful (are the) connections it could make with other mathematical ideas and

> situations…Hence, it is very important to encourage learners to reflect on actions they make in order to be able to perceive mathematical processes as objects."(p. 3)

They further advocate,

> "Every student should be given an opportunity to play with manipulatives. Just a demonstration by a teacher is not sufficient to realize their full potential and not in line with the theoretical rationale of their usage since they are meaningful to the extent they involve interactive activities."(p. 4)

Citing Suydam & Higgins (1976), Durmus & Karakirik concur that

> "Manipulative materials should be used in conjunction with exploratory and inductive approaches" (p. 4)

and conclude that

> "Most manipulatives in mathematics simply implement the 'learning ***with*** model' approach. However, educators also need to consider the possibility of designing manipulatives employing the 'learning ***to*** model' approach since full potential of any technological device could be achieved through its usage as a communication tool to model the concepts and relations at hand." (p. 6) (Italics and bolding added.)

Clearly, more research is needed to determine the appropriate role of virtual manipulatives, in both mathematics teacher preparation and pre-K-12 mathematics instruction. The educational impact of the ever-burgeoning accessibility and advancement of technology demands the attention of mathematics educators. With the Georgia Performance Standards as our springboard, Georgia teacher educators are in a position to influence the preparation of internationally competitive mathematics students. Will the use of virtual manipulatives contribute to our progress? It's an important question that begs an answer.

Final Comments: Plans for Future Research

Faculty at Clayton State University plan an investigation of the effectiveness of both concrete and virtual manipulatives will take place in CSU's Number Concepts and Relationships class fall 2008. Approximately 25 juniors in CSU's Middle Level Teacher Education program will participate in the investigations. They will have access to concrete manipulatives, including fraction circles, pattern blocks, decimal mods, and two-color counters. They will also have laptop computers and Internet connections, giving them access to virtual manipulatives websites and applets. Lessons will address Georgia Performance Standards number concepts for grades 6 – 8, including the properties and arithmetic of rational numbers, integers, and prime and composite numbers. Students will work in groups of 2 or 3, using concrete manipulatives to build conceptual understanding of number concepts. Each of these experiences will be followed by similar activities, replacing the concrete manipulatives with virtual manipulatives. Communication and reflection will be incorporated throughout and feedback will be gathered from the students.

Assessment of the effectiveness of the various manipulatives will include observation, written feedback, performance tests, and tasks similar to those advocated by Ball et al. (2008). Students will also study the research on the effectiveness of manipulatives. Statistical methods will be used to investigate possible correlations among students' characteristics and the effectiveness of the different types of manipulatives, etc. Details of this work will be presented at the GAMTE conference in October.

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Actively Engaging Pre-service Teachers in Geometry and Measurement

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Abstract

Current work in mathematics education suggests that the learning experiences in which teachers engage during undergraduate study influences their knowledge of and beliefs about mathematics and the ways in which they will teach (Allen, et. al., 2008; CBMS, 2001; Hill, Rowan, & Ball, 2005; National Research Council, 2001). However, very little is known about pre-service teachers' learning experiences and how those experiences influence their thinking about mathematics teaching and learning. The classroom excerpt described here attempts to illuminate how pre-service, elementary teachers' active engagement in the learning of geometry and measurement influences their mathematical power: a positive disposition toward mathematics, ability to reason about mathematics, facility in making connections across content strands and to other subjects, and proficiency in communicating mathematical ideas (Baroody & Coslick, 1998; National Council of Teachers of Mathematics, 1989; Orrill & French, 2002). The author calls for research that more closely examines students' learning experiences and educational outcomes such as mathematical power and mathematics knowledge for teaching (Hill, Rowan, & Ball, 2005).

Actively Engaging Pre-service Teachers in Geometry and Measurement

Actively engaging students in the learning process is a central feature of many efforts to reform the teaching and learning of undergraduate mathematics (Bryant, 1998; Kvam, 2000; Millett, 2001; Roddick, 1997; Treisman, 1992). The Mathematics Association of America (MAA) developed *Quantitative Reasoning for College Graduates: A Complement to the Standards* (1998) to address concerns about the types of mathematical experiences that all undergraduate students should have as they develop quantitative literacy. In this report, the MAA advised that traditional lectures be replaced with more active, engaging experiences that require students to engage in teamwork, discussion, and writing about mathematics. In their recommendations for preparing K– 12 mathematics teachers, the Conference Board of the Mathematical Sciences (CMBS; 2001) also discusses the importance of actively engaging pre-service teachers in the learning process. They identified active involvement as a goal of elementary and secondary mathematics education and claim that in order for teachers to actively involve students in their own classrooms they need to have similar experiences in their college mathematics courses. While there is no agreed upon definition of active learning, scholars assert that active learning involves talking, listening, writing, reading, and reflecting (Hobson, 1996; Meyers & Jones, 1993; Yackel & Cobb, 1996). These elements all engage the brain in different thinking processes or operations that lead to the creation of new mental structures, and thus, are elements of active learning.

In addition to engaging in active learning experiences, teachers of mathematics also need to develop mathematical power so they can foster its development in their students (Baroody & Coslick, 1998). Scholars suggest that there is something important about having an understanding of and a positive disposition toward mathematics, being able to reason about mathematics, make connections across content strands and to subjects outside of mathematics, and being able to communicate mathematical ideas (Baroody & Coslick, 1998; National Council of Teachers of Mathematics, 1989; Orrill & French, 2002).

Restructuring a Geometry and Measurement Course

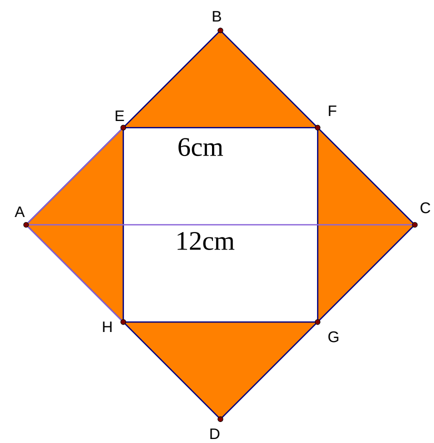
As I prepared to teach an undergraduate Geometry and Measurement course for pre-service elementary teachers in the spring of 2008, I sought ways to create a learning environment where students would have many opportunities to learn by actively engaging in the study of mathematics and that would foster the development of their mathematical power. The course was structured with these goals in mind and incorporated the following features:

- Students sat with and regularly worked in pairs or small groups to provide them with opportunities to communicate about mathematics with each other and to share and compare ideas and problem solving strategies.
- Students were encouraged to ask questions (of the teacher or of other students) or make propositions or conjectures about mathematical ideas as they arose. Questions, propositions, and conjectures were then addressed and explored by the class.
- Students were always required to give explanations of their thinking and reasoning processes as to illuminate the process of developing mathematical ideas and solution strategies.
- Students were regularly asked to consider and identify different ways of thinking about a problem and to explore multiple strategies for solving a problem.
- Students were consistently asked entrance and exit questions that were used throughout the semester to assess students' ability to articulate their conceptual understanding of geometry and measurement in writing and their ability to make connections among different concepts.

- Students were frequently given think-pair-shares or think-write-pair-shares so as to promote opportunities for independent thinking and formulation of ideas before sharing those ideas with other students or with the class.
- Students, as a class, periodically worked through elementary-level problems followed by watching and discussing video clips of elementary students working on those problems and developing their understanding of mathematical concepts.

An Excerpt

It was the last day of class, before the start of spring break. Some of the 21 students were getting restless and wanted to begin spring break as soon as possible. About two-thirds of the way through the class, we were finishing a word problem when I heard Jason (pseudonym, as are all proper names) whisper that he was ready to leave. I thought for a moment about his request and then reminded the class about a problem that I had written on the board at the beginning of class. It was an interesting problem that I told them I wanted to make sure we got to before the end of class. Up to that point, no one had considered the problem, but as some students were beginning to get antsy, I proposed that once a student solved that problem, he or she could leave. The problem was to find the area of the shaded region given square EFGH with segment EF=6cm inscribed by square ABCD with diagonal AC=12cm. It should also be noted that the inner square was inscribed by the outer square using the midpoint of the sides of the outer square.



The students immediately started working on the problem, and although Jason was the first to run up to show me his solution, no one stopped working because it had *already been solved*. Perhaps the students were anticipating my usual questions: "Ok, so what did you do and how did you get this as your answer?" and "Can you find another way to solve it to verify that your solution is correct?" This questioning, of course, was not explicit in the deal "If you solve it, then you can leave," but the students had enough experiences in the course to anticipate this expectation and recognized that thinking about a mathematics problem in a different way can help them make sense of their solutions.

Jason's First Solution:

Jason: "Is this the answer?" [he showed me a sketch of the problem and solution $A=36cm^2$]

Me: "Hmmm, how did you get that?"

Jason: "Well, the area of this triangle [triangle EFB] is $9cm^2$ and there are 4 of them, so the total area is 36 cm^2."

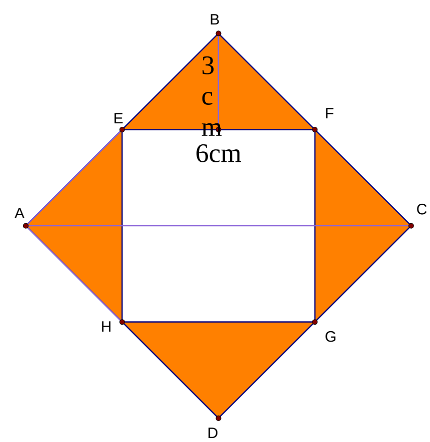
Me: "How did you calculate the area of the triangle?"

Jason: "You know the base is 6cm 'cause that is given in the problem, and the height is 3."

Me: "How do you know that the height is 3?"

Jason: "Because…, um, wait…I've got to look at that."

Me: "Ok." [Jason headed back to his seat]



Jason was the only male in the class. He was formerly a secondary mathematics education major and has taken more advanced math courses than most of the other elementary education majors in the class. He was fairly confident in his ability to do mathematics, particularly when it involved calculations. He, however, had been a bit humbled by our study of the underlying mathematical ideas involved in elementary school mathematics along with his discovery of the value in considering different ways to think about and solve problems. Jason had come to respect his classmates' perspectives and often commented, "I never would have thought of solving it that way."

As Jason walked past Laverne's table, she looked up from her work at me, "Are you going to show us how to do this?!" She was frustrated. Laverne was always frustrated. She was a non-traditional student who had very little confidence in her ability to do mathematics. She often struggled with trying to get started on a problem, mostly because she questioned herself and whether or not she was doing the problem correctly. Whenever she was asked what she has a question about, Laverne responded, "Everything!" For the unit on measurement, she had been sitting at a table with Ursula. Ursula did not say much in class unless she was asked, but she worked well with Laverne. Ursula was also very willing to begin working on something even when she was not quite sure about what to do. She was comfortable figuring things out and adjusting her solution along the way. When Laverne asked for my assistance I asked her what she had already tried; as she was gearing up to tell me that she was completely lost, Ursula tapped her to get her attention. Laverne turned her attention to Ursula and the two began discussing how to solve the problem.

A moment later, Jason came back up to explain his solution. I noticed while he was working with his group he seemed to be explaining his way of solving the problem and his group members were listening and asking questions about why his solution made sense.

Jason's Second Solution:

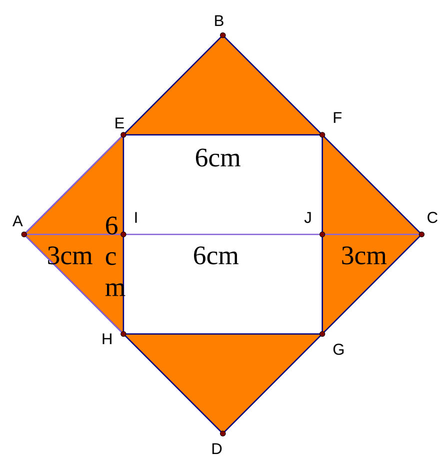
Jason: "Ok. I just had to go back and make sure what I did made sense."

Me: "Ok. Let's see. What did you do?"

Jason: "So if this side is 6 [refers to segment EF] and this distance is 12 [segment AC], then since they are both squares, this is 3 [segment AI] and this is 3 [segment JC]."

Me: "Ok."

Jason: "So the base of the triangle [segment HE of triangle AHE] is 6cm and the height (segment AI) is 3cm so the area is 9 [pointed to his written work ½*6*3]. There are 4 triangles, so the total area is 36 cm^2. Is that right?"



Me: "That's interesting. So can you find another way to verify that it is correct?"

Jason: "So you want me to do it another way? Ahh!"

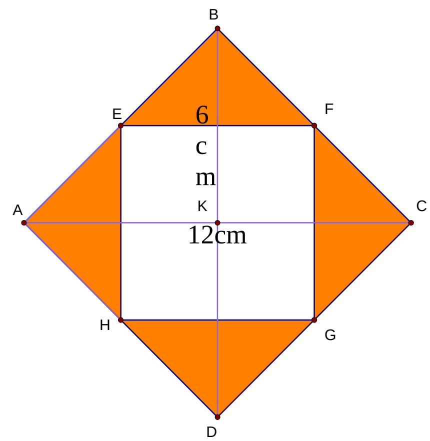
Me: [I smiled. He seemed to need some encouragement.] "Yes. Think about our general strategy, figure out the area and subtract the parts you don't want."

Jason: "Ok." [He headed back to his table and began discussing the problem with his group members again. This time he appeared to be listening as his group members' explained their ideas.]

Abigail, Ursula, and Laverne soon came up excitedly to show me their solutions. Abigail is a non-traditional student who does not recall ever taking geometry in high school and sometimes has trouble understanding what problems are asking. Throughout the class she enjoyed using different learning tools to model problems so that she could understand what she was really trying to find. She is very careful when drawing shapes and often uses graph paper to help with her precision. The graph paper helped her to see more clearly how to solve this problem.

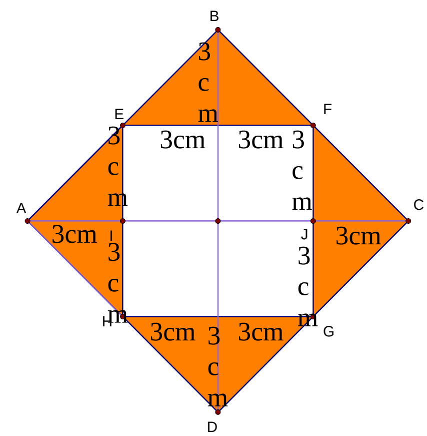
Abigail's Solution

Abigail noted that because ABCD is a square, segment AC bisects segment BD so the distance from point B to the center of square EFGH [point K] is 6cm. She then solved the problem by looking at congruent triangles ABC and ADC, found the area of these triangles, and then subtracted out the area of square EFGH: $A=2(1/2)(12)(6) – (6)(6) = 36\ cm^2$.



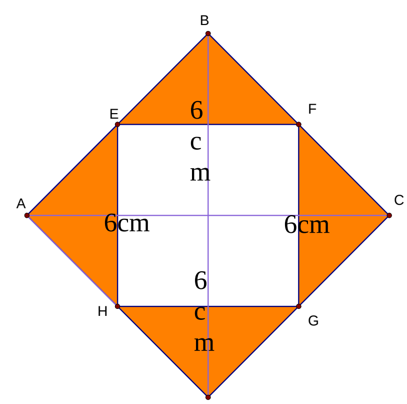
Laverne and Ursula's Solution

Ursula listened carefully to Abigail's solution, but Laverne could hardly contain her excitement. She interrupted to say that she and Ursula thought about it that way too, but ended up breaking it up into smaller triangles. As soon as Abigail finished, Laverne proudly explained that they found eight congruent right triangles each with a base of 3cm and a height of 3cm, and then calculated their areas: $A=(8)(1/2)(3)(3) = 36\ cm^2$. When I asked Ursula and Laverne how they figured out that each of these measurements is 3cm, Ursula explained that if segment AC is 12cm and segment IJ is 6cm then you are left with 3cm on either side of points I and J so segment AI is 3cm and segment JC is 3cm. She told me that they know this is true because both ABCD and EFGH are squares and E, F, G, and H are midpoints. Ursula also noted that segment BD bisects segment AC and it also bisects segment EF, so you end up with eight right triangles each with a base of 3cm and a height of 3cm. I followed up with a couple questions directed to Laverne so I could verify that she understood what Ursula was explaining. She appeared to understand, so I sent them off to solve the problem another way.



Jason's Group's Solution

Jason and his group worked out another solution and he came back up to share it with me. They had sketched in segments AC and BD and noted that they bisect each other. He explained that there are four congruent quadrants, each was a right triangle with area $=(1/2)(6)(6)=18\ cm^2$. So the area of square ABCD is $4(18)=72cm^2$. Then you have to subtract off the small square EFGH: $A=72-(6)(6)= 36\ cm^2$. I smiled and wished Jason an enjoyable spring break – ten minutes before class was scheduled to end. Instead of rushing out the door, Jason went back to his table to discuss the problem with his group some more. I guess he wasn't in such a rush after all.



Discussion

Proponents of active learning argue that new ideas are formed and discoveries are made in mathematics not by individual competition but through collaboration with colleagues, and this process is central to the nature of the subject (Mau & Leitze, 2001; Rogers, 1992; Yackel & Cobb, 1996). Rogers (1992) states

> A pedagogy that emphasizes product deprives students of experiencing the process by which ideas in mathematics come to be and perpetuates a dualistic view of mathematics in which right answers are known by authorities and are the property of experts. Such a pedagogy strips mathematics of the context in which it was created and is based on misconceptions about its very nature. (p. 42)

Mau and Leitze (2001) add, "When we teach our mathematics students to 'be quiet and listen,' we deprive them of the opportunity to create their own meaning, disempower them, and remove their opportunity to develop autonomy" (p.38). The lack of opportunity to learn can be detrimental to students' mathematics achievement in general (National Research Council, 2001). For women in particular, robbing them of the opportunity to engage in the learning process inhibits the development of their voice in the learning of mathematics and further marginalizes them in an already male-dominated discipline (Mau & Leitze, 2001; Rogers, 1992).

This excerpt provides some evidence of and suggests that by actively engaging students in doing mathematics, teacher educators can foster the development of mathematical power: a positive disposition toward mathematics, ability to reason about mathematics, facility in making connections across content strands and to other subjects, and proficiency in communicating mathematical ideas (Baroody & Coslick, 1998; National Council of Teachers of Mathematics, 1989; Orrill & French, 2002). Mathematical power could potentially influence the development of a teacher's ability to draw upon and utilize other important tools for teaching, such at mathematics knowledge for teaching (Hill, Rowan, & Ball, 2005; Hill, Schilling, & Ball, 2004). Presently, no studies exist that have examined pre-service teachers' mathematical power or its development during undergraduate study. Consequently, there is no information on relationships between mathematical power and other important educational outcomes. There is a need for research that provides a better understanding of the types of learning experiences that teachers should have during their teacher preparation programs.

Reports on the status of mathematics education in K–12 schools express concern about teacher preparation programs and the types of opportunities made available for pre-service teachers to learn important mathematics in meaningful ways and call for more research to understand this segment of mathematics education (CBMS, 2001; National Mathematics Advisory Panel, 2008). Teacher educators have a responsibility to address recommendations put forth by national, state, and local organizations and must continue to seek ways that create opportunities for future teachers to learn mathematics for teaching. I propose that one area that deserves further examination is the relationship between active learning opportunities and the development of mathematical power. Findings from a study that examines these components of pre-service, elementary teachers' study of mathematics is forthcoming.

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The PAAPS Strategy for Teaching Mathematics Content

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Abstract

In the fall of 2005, I started teaching the mathematics content course Algebra and Geometry for Teachers. The majority of students in the course are pre-service middle school teachers. Instead of teaching the course by demonstrating rigorous proofs, I wanted to use teaching strategies that would build the students' content knowledge and connect to their roles as future mathematics teachers. I chose to make problem solving a focal process standard by having students problem-solve for a majority of classroom time. In addition, the students complete a major project entitled "Provide, Attempt, and Assess Problem Solving" or PAAPS. For PAAPS, each student provides a non-routine algebra, geometry, or analytic geometry problem to five of their classmates. Each student then attempts the five problems received, and returns the attempted problems to be assessed by the student who provided them. In this article, I share the results of using PAAPS as evidenced by student surveys and student work (problems chosen, problems worked, and problems assessed). Included in the surveys are the mathematical and pedagogical ideas that the students reportedly learned.

Background

In the fall of 2005, I started teaching the mathematics content course Algebra and Geometry for Teachers, which is comprised almost entirely of preservice middle school teachers. I did not want to teach the course by only demonstrating rigorous proofs in the classroom, so I decided to redesign the course. I wanted to use teaching strategies that would build the students' content knowledge and connect to their roles as future mathematics teachers. I decided to have class meetings comprised of both traditional class lecture sessions and problem-solving sessions. The class lectures included direct instruction, which sometimes included rigorous proofs. In the problem-solving sessions, students worked on non-routine algebra and geometry problems in groups of four to five.

As I reflected upon my experiences using a problem-solving approach with preservice middle school mathematics teachers in another course, I realized that I needed to make some changes. While the problem-solving activities were well-grounded in terms of both "real-world" contexts and mathematical concepts that were appropriate to middle school mathematics, many students struggled with their attitudes toward problem-solving. Several students did not see why problem-solving was useful. They decided it was better for teachers to "just show students what they need to know." While I knew that the students learned a lot of mathematical content in the problem-solving sessions, several did not appreciate its usefulness. As a teacher, I considered myself a facilitator, but some students did not consider this "teaching." Many students did not see the problem-solving sessions as relevant to their roles as future mathematics teachers. Rather than watching me in the role of teacher in a problem-solving classroom, the students needed to be in that role themselves. The students needed to experience activities that make these connections explicit.

Development and Description of the PAAPS Teaching Strategy

While the students learned from the problem-solving sessions, they would have a more positive learning experience and they would be more motivated to problem solve if they could see the usefulness of the experience for their role as future teachers. As I thought of potential strategies to accomplish this, I remembered a strategy that a former colleague, Karen Cohen at the University of Florida, uses with one of her courses. Karen has students "prepare a problem to stump their peers." The students share their problem with other students and assess the work of the other students. Karen's course, however, is different from mine in two ways that concerned me. First, her course is populated with graduate students, while mine is populated with junior undergraduates. Second, her course is a methods course, while my course is a mathematical content course. The potential benefits of using this teaching strategy, however, alleviated my concerns.

Cohen's teaching strategy focuses on the foundation of facilitating problem solving: (a) finding good non-routine problems for the students to work, (b) working non-routine problems to ensure they are appropriate for the students, and (c) assessing students' problem solving. I developed a three-part at-home project for students, entitled "Provide, Attempt, and Assess Problem Solving," or PAAPS, to reflect this focus. PAAPS is a three-week project, with one week given for each of the three parts of PAAPS. To start PAAPS, students are randomly placed in groups of six. In part one of PAAPS, each student independently finds a non-routine algebra, geometry, or analytic geometry problem for the other students in the group to work. The student who found the problem successfully works their chosen non-routine problem, using as many different strategies as possible. On the due date, each student provides his/her problem, without solutions, on a sheet of paper to the other members of his/her group. Also, the student provides the professor with a copy of his (or her) chosen problem along with a worked-out solution(s).

In part two of PAAPS, each student works the five problems that were furnished by the other group members. On the due date, the students return the five attempted problems to the group member that provided the problem. In addition, each student gives the professor a photocopy of each attempted problem.

In part three of PAAPS, each student assesses his/her five classmates' work on the problem provided. When assessing the work, they must: (a) provide a numerical score on the paper, giving partial credit for the work; and (b) provide written feedback on the paper, including proper mathematical content and process language in the writing. In addition, the students write a cover sheet to turn in to the professor. On the cover sheet, they describe the: (a) technique used to assign a numerical score to the papers, (b) mathematical ideas learned from the project, and (c) teaching ideas learned from the project. On the due date, each student returns a copy of the assessed papers to their group member that worked the problem. In addition, he or she gives the professor a copy of the papers assessed with a cover sheet stapled on top.

When I assigned a grade to the students' work on the PAAPS project, I created a scoring rubric based on what I requested the students do on the project description handout. I scored them based on whether or not they completed what they were asked to do as well as their effort on the tasks. See Appendix A for a sample scoring rubric.

Analysis of Student Work: Provide, Attempt, Assess Problem Solving

I used the PAAPS teaching strategy during the fall of 2005, and devised techniques to document its use for the fall of 2006. During the fall of 2006, there were 19 students in the class, who provided 18 unique problems (one problem was duplicated). Of the 19 students in the class, 63% $\left(\frac{12}{19}\right)$ successfully worked their chosen problem, 21% $\left(\frac{4}{19}\right)$ attempted but did not successfully work their chosen problem, and 16% $\left(\frac{3}{19}\right)$ turned in a copy of a solution from an external source (internet or text) and did not attempt to work their chosen problem themselves. Appendix B includes a selection of the problems chosen by students.

For part 2 of PAAPS (attempt problem solving), I only assess whether the students make a good effort on the five problems that were provided to them by their classmates. Eighty-nine percent $\left(\frac{17}{19}\right)$ of the students made a good effort on all (five) of the problems provided to them. Of the remaining two students, one student did not attempt one of the problems, and the other student did not attempt four of the five problems.

There were several commonly used comments in the students' written feedback when they assessed the other students' problems. Virtually all of the students used comments such as "Great job,", "You got it right," "This is a good start," and "Great strategy." Overall, the students did a very good job of promoting confidence with positive feedback. Other commonly used comments, such as "Start the problem by…" and "Show how you got your answers" gave direction on how to start the problem-solving process and how to properly communicate that process. Several students noted in their feedback that the problem-solver used a different strategy from their own, with comments such as "This is a good idea. I did not think of it."

Analyzing the quality of student feedback revealed 21% $\left(\frac{4}{19}\right)$ of the students wrote "good" to "excellent" feedback on the papers that they assessed. For example, Melanie's feedback "You found the area of one rectangle, now find the dimensions" was categorized as excellent feedback. Max's feedback "Find the five main ways to get 11 points, then use those ways to get other combinations. There are 19 combinations." was categorized as good. Twenty-six percent $\left(\frac{5}{19}\right)$ of the students wrote "average" feedback comments. For example, Brandon did an excellent job of writing feedback on papers with strategies that were different from his, as he properly assessed Caleb's solution, which used the Law of Cosines. However, for the incorrect solutions, his feedback was poor since he did not give feedback to help students toward a correct solution. In particular, for one student with an incorrect strategy, Brandon did not write any comments. For another student with an incorrect strategy, Brandon wrote "Great deduction!" Thirty-seven percent $\left(\frac{7}{19}\right)$ of the students wrote feedback that was "fair" or "poor." Sixteen percent $\left(\frac{3}{19}\right)$ of the students only wrote feedback such as "good job" since all students worked their problems correctly. These students' feedback was categorized as "none."

Of particular note are the students whose feedback was "poor" $\left(\frac{6}{19} \approx 32\%\right)$. All three of the students who copied their solutions (from the internet or a text) in Part One of PAAPS rated "poor" in the quality of their feedback. For example, in Part One, Carl turned in a copy that described the box as being inscribed in a circle of radius r with all 8 corners of the box touching the surface of the sphere. However, he left this information out when he provided the problem to his classmates to work, so there was not enough information for students to obtain a solution. In his feedback, he never realized that his stated problem was incomplete. He simply wrote the text solution that he copied, no matter what the problem-solvers wrote on their papers. Similarly, Jay simply wrote "wrong answer" and "no work" for his feedback, while Leigh wrote feedback that was centered on her copied solution, and did not build well on what her classmates wrote. For example, Caleb wrote in his solution to Leigh's problem (See Appendix B for problems), *d = outer edge – inner edge = 2π (r + 5) - 2πr = 10π meters*. Caleb then stopped working the problem. Leigh wrote the feedback $d = \frac{2\pi r}{\pi} = \frac{2\pi 5}{\pi} = 10\, m/s$, which was a formula in her solution and did not connect well to Caleb's work.

Lisa's feedback was also rated "poor." In Part One of PAAPS, her solution to the problem that she chose was incorrect since she based all of her work on the assumption that $\overline{AF} = \overline{FC}$ = 5 ft. This gave her the incorrect height $\overline{EF}$ = 5.15 ft. In Lisa's assessment of six classmates' work, two of the six, Holly and Ashley, had the correct height $\overline{EF}$ ≈ 6.8 ft. Holly used strategies similar to Lisa, first the Pythagorean theorem to find $\overline{AB}$ and $\overline{CD}$, then noted that triangles AEB and CED are similar to set up the ratio $\frac{\overline{AE}}{15 - \overline{AE}} = \frac{\overline{AB}}{\overline{CD}}$ and find $\overline{AE}$. Holly then used the ratio for the similar right triangles $\frac{\overline{EF}}{\overline{CD}} = \frac{\overline{AE}}{15}$ to find the desired height $\overline{EF}$ ≈ 6.8 ft. Lisa wrote on

Holly's paper "your work is clear and your answer is right," so perhaps Lisa caught her own mistake from her previous solution of $\overline{EF}$ = 5.15 ft. Ashley started the problem in the same way as Lisa, using the Pythagorean theorem to find $\overline{AB}$ and $\overline{CD}$. After this, Ashley used a different strategy. She treated point A as the origin on the coordinate system. She then found the equations of the lines $\overleftrightarrow{AD}$ and $\overleftrightarrow{BC}$ in slope-intercept form y = mx + b. Lastly, she set the two equations equal to determine their point of intersection, with the y-coordinate of this intersection giving her the height $\overline{EF}$. Lisa, however, did not see this as an acceptable strategy, writing the comment on Ashley's paper "AEF and ADC are similar triangles. See that $\frac{EF}{CD} = \frac{AE}{AD}$, continue with this and you will get the answer."

Analysis of Cover Sheet Survey

When asked to identify the mathematical ideas that they learned from the PAAPS project, the major theme in student responses was that there are many different ways to solve a mathematics problem. In addition, some students focused on specific mathematical ideas that they learned, such as Ellen's statement, "I learned geometry. Within geometry, I became more familiar with topics such as area, volume, perimeter, 2-D and 3-D figures, and induction and deduction." Other students stated that they "refreshed their memory" on topics, such as the Law of Cosines. Some students focused on the process of doing mathematics. For example, Heather stated, "I learned many mathematical ideas from this assignment. One idea was how to change from one type of problem to another. Each problem was very different and required different ways of approaching it. It was very helpful to learn how to change mentally to look at different problems."

There were also several themes identified in the teaching ideas students reported learning from PAAPS. In particular, the students reported they learned: (a) to provide problems that include clear and concise directions; (b) to provide fun, challenging problems to children that are centered on a certain concept so they will be more likely to want to learn how to solve [problems] and understand [concepts]; (c) to ensure students see different ways to solve a problem; (d) teachers need to be knowledgeable of different strategies because students work problems in different ways; (e) assigning points (grades) can be difficult; (f) how to assess someone else's work and how to better understand their thinking; and (g) feedback to the student is important. In relation to feedback, one student wrote, "If they are never told what they did wrong and what they can do to improve, then they are not being given the proper chance to improve."

Conclusion

According to *The Mathematical Education of Teachers* (CBMS, 2001), preservice teachers need to: (a) acquire a deep understanding of mathematics, (b) learn to pose good mathematical questions, and (c) look at problems from different perspectives. A deep "mathematical knowledge for teaching…allows teachers to assess their students' work, recognizing sources of student errors and the students' understanding of the mathematics being taught" (CBMS, 2001, p. 13). According to *The Professional Standards for Teaching Mathematics* (NCTM, 1991), mathematics teachers' education should enhance knowledge of: (a) ways to reason and communicate mathematically, (b) multiple representations of mathematical concepts, and (c) means for assessing student understanding of mathematics. In addition, it should help one "develop a sense of self" as a teacher of mathematics (NCTM, 1991, p. 161). The PAAPS teaching strategy is designed to facilitate these aspects of the preservice teachers' preparation.

The PAAPS process allows students to find problems that interest them and work the problems furnished by their peers. In addition, they see their chosen problem worked by five other students, so most of the students are challenged to think about their chosen problem more deeply. These six problems provide a rich experience. Many students turn in excellent attempts at problem-solving both on their own problem and the problems of others (see the strategies of Holly and Ashley on Lisa's problem that is previously discussed). The students wrote numerous comments concerning teaching ideas learned. This demonstrates that PAAPS was successful at highlighting the relevance of problem solving to their roles as future teachers.

The PAAPS teaching strategy facilitates mathematical learning by promoting problem solving as a valid mathematical endeavor. In addition, students were able to refresh their memory on some mathematical ideas, and learn about other mathematical ideas. The PAAPS teaching strategy clearly demonstrates to students that there are many ways to solve problems, and exposes them to multiple representations.

The PAAPS process allows students to make mistakes and learn from them. Some students made a mistake when they worked their own chosen problem, but then found their mistake when they assessed the work of their classmates. Brian chose his problem "simply because it fell under the geometry category on the *Figure This* web site. When asked what teaching ideas he learned from PAAPS, Brian stated that he needed to be more careful in his problem selection.

In addition to making discoveries on their own, I critique each student's work. This provides the opportunity to further guide their thinking. Students receive comments such as, "Guide your students on what to do next," "Provide written feedback," and "Carefully consider solutions that are different from yours." For students who simply copy the solution of their chosen problem from a text or the internet, I write, "You need to work the problem yourself so that you can properly assess the work of others." I comment on how this affects assessment of their classmates.

After the students turned in their work for part three of PAAPS (assess problem solving), the project was concluded, other than receiving their grades for the project. In the future I would recommend three additions to PAAPS to enrich the mathematical content knowledge of the students. First, have the students meet with their group when they return the assessed problems. Ask each student to mathematically discuss and justify their solution to each problem and have each student justify the scores given for each problem. Second, share all of the problems that fit within the targeted category (algebra, geometry, analytic geometry) with the class. This will allow all students to work all of the problems provided by the class, thereby increasing the learning experience for them as well as providing a means to assess their mathematical understanding of the problems at a later date. Third, conduct the project more than once. In this way, one can assess improvement in students' ability to provide, attempt, and assess problem solving.

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Appendix A
Sample Scoring Rubric

Part 2 - Attempt 5 Problems from 5 Classmates (20 points total)

Problem Solving Effort (10 points possible)

0, 1, or 2 points assigned for each of the 5 problems as follows:

0 The student did not attempt the problem or did not work the problem himself or herself.
1 The student attempted the problem (him/herself), but there is not a clear process.
2 The student attempted the problem (him/herself), and there is a clear process. If the problem is a multi-step problem, then the “clear process” may only be the first step of the multi-step problem. The strategy may or may not be a strategy that leads to a correct answer.

Assignment Requirements Met (10 points possible)

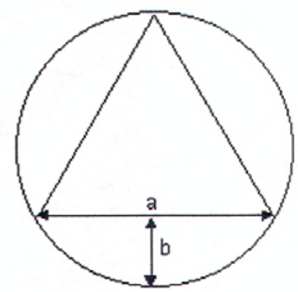
Assign the points below if the listed requirement was met. If it was not met, then assign 0 points.

4 Turned in two copies of each problem attempted.
4 Returned all of the worked problems to the group members who provided them.
2 Collected problem (chosen in Part 1) from each group member who worked it.

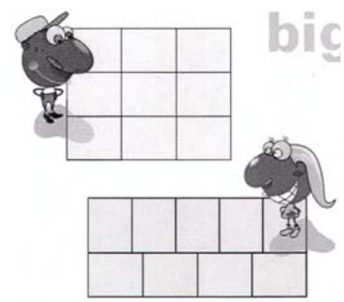
Appendix B

Problems Chosen by Stud

1. Brandon: In the picture is an equilateral triangle inscribed in a circle. Given that the radius of the circle is 1, what is the length of a and b? Stated source: http://mathproblems.info

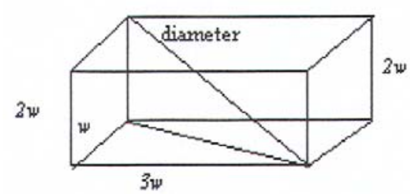


2. Melanie: Helix and Polygon both used the same number of identical concrete pieces to make their patios. The area of each patio is the same: 180 square meters. What are the dimensions of a single piece of concrete? Stated source: www.figurethis.org

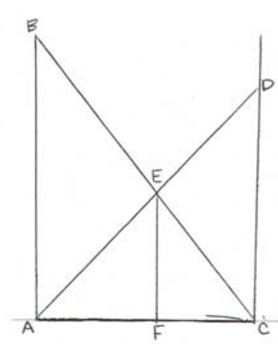


3. Leigh: If a circular track is 5 meters wide and it takes a horse, traveling at its fastest speed, π more seconds to travel the outer edge than the inner edge, what is the horse's speed? Stated source: www.hawaii.edu/suremath/jhorseRace.html

4. Carl: Find the width of the box *w*. Stated source: none



5. Lisa. Two walls are 10 ft. apart. Two ladders, one 15 ft. long and one 20 ft. long are placed at the bottom of the walls, leaning against the opposite walls. How far from the ground is the point of intersection? Stated source: www.mathforum.org



Preservice Teachers' Disposition Self-Appraisals: Is There a Connection to Mathematics Instructional Practices?

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Abstract

This paper explores preservice teachers' dispositions and the connection to mathematics instructional practices. The Theory of Culturally Relevant Pedagogy served as the theoretical grounding for the study. Hence, its three broad propositions: conceptions of self and others, social relations, and conceptions of knowledge structure the argument presented. The participants in the study are elementary preservice teachers enrolled in a post-baccalaureate alternative certification program. The full-time, accelerated program focuses on and is specifically designed for individuals committed to teaching in urban, high-poverty schools. A dispositions survey, open-ended narratives, and teaching observation rubric served as data sources for the qualitative study. Findings indicate a connection between dispositions and instructional practices; however, nuances existed between the components of culturally relevant pedagogy.

Introduction

The field of psychology has long been engaged in unlocking the complexities of the human mind. In recent years, however, education scholars have attempted to understand the power of thought and its implications for teaching and learning. One might argue that the mind is the most prevailing facet of a human being. As the United States and its schools become more diverse, the notions that teachers hold about various student demographic groups becomes increasingly significant. The U.S. Census Bureau projects that by the year 2042 Whites will no longer make up the majority in the U.S. They estimate that by 2050 Whites will comprise 46% of the population, Hispanics 30%, African Americans 15%, and Asians 9%. The National Center for Education Statistics indicates that in 2003 U.S. public elementary and secondary schools were 58.7% White, 17.2% Black, and 18.5% Hispanic. In contrast, its teachers were 83.1% White, 7.9% Black, and 6.2% Hispanic (Strizek et al., 2006). These statistics highlight the fact that most Black and Hispanic children will be taught by a teacher that does not share their racial/ethnic background. In addition to being racially and culturally diverse, U.S. public schools are also linguistically and socioeconomically diverse. Nineteen percent of students speak a language other than English at home (Rooney, 2006). Further, the U.S. Census Bureau data reveals that 18% of children under eighteen years old are living in poverty.

With such demographics it is critical that preservice teachers be adequately prepared to provide effective, high-quality mathematics instruction to all students regardless of their race, ethnicity, gender, socioeconomic status, culture, or other characteristics. Now, more than ever equity in mathematics education is critically salient. Teachers must function as agents of change and challenge the pervasive societal belief that only some students are capable of learning mathematics. They must hold high expectations for all students and challenge the implicit, often unspoken, notion that only the experiences of some students are valuable and reflect mathematical knowledge. The lived experiences, prior knowledge, intellectual strengths, and personal interests of all students should be valued and utilized as a springboard for learning. Therefore, mathematics teachers must reconceptualize the nature of students' mathematical knowledge, bridging informal and formal mathematical experiences, and providing students opportunities to demonstrate their understanding of mathematics in a multitude of ways. If these goals are to be met, it is important that teachers hold excellence-oriented views toward all students and those views are explicitly reflected in all aspects of the mathematics teaching and learning process – instructional planning, decision making, practices, and classroom community.

Given the critical significance of reaching all mathematics learners, this study sought to examine the connection between preservice teachers' disposition self-appraisals and their mathematics instructional practices. In essence, are the views teachers express congruent with what they do in the classroom?

Theoretical Framework

The Theory of Culturally Relevant Pedagogy (Ladson-Billings, 1995) served as the theoretical grounding for framing elementary preservice teachers' self-appraisals of their dispositions. Ladson-Billings asserts that culturally relevant teachers exhibit the following broad qualities with respect to the underlying propositions: (a) Conceptions of self and others suggests that culturally relevant teachers hold high expectations for all students and believe all students are capable of achieving academic excellence; (b) social relations infers that culturally relevant teachers establish and maintain positive teacher-student relationships and classroom learning community as well as are passionate about teaching and view it as a service to the community; and (c) conceptions of knowledge suggests that culturally relevant teachers view knowledge as fluid and facilitate students' ability to construct their own understanding. Ladson-Billings' work provides a context for illuminating instructional practices that facilitate the academic success and cultural competence of traditionally underserved student populations. Hence, it is salient for the goal of the study.

Methodology

The fourteen participants in the study are elementary preservice teachers enrolled in a post-baccalaureate alternative certification program. The full-time, accelerated program focuses on and is specifically designed for individuals committed to teaching in urban, high-poverty schools.

A dispositions survey, open-ended narratives, and teaching observation rubric served as data sources for the study. All three sets of data were collected during the same semester. The dispositions survey utilized in the study is a component of the program assessment preservice teachers complete at the end of their program. The disposition items on the survey are based on the Interstate New Teacher Assessment and Support Consortium (INTASC) Standards. Preservice teachers are asked to self-rate their demonstrated level of knowledge on a scale of 1 to 5 with 1 being not demonstrated; 2 - basic; 3 - developing; 4 - proficient; and 5 - advanced. For this purpose of this study the disposition items were categorized and analyzed with respect to the broad propositions of culturally relevant pedagogy: conceptions of self and others, social relations, and conceptions of knowledge. See table 1.

Table 1. Disposition Items aligned with Culturally Relevant Pedagogy

Proposition: Conceptions of Self and Others			*Rating*		
	Not Demonstrated 1	Basic 2	Developing 3	Proficient 4	Advanced 5
I value the individual differences among all students and accommodating these differences to fulfill the learning needs of all students.					
I value human diversity, respect students' varied talents and perspectives, and show sensitivity to community and cultural norms.					
I value personal reflection in my development.					

Proposition: Social Relations			*Rating*		
	Not Demonstrated 1	Basic 2	Developing 3	Proficient 4	Advanced 5
I value an environment in which students have clear expectations and in which time and space in the classroom are managed to ensure student engagement.					
I value the many ways in which people seek to communicate and values responsive listening.					
I value relationships with school colleagues, parents, agencies, and the community.					

Proposition: Conceptions of Knowledge			*Rating*		

	Not Demonstrated 1	Basic 2	Developing 3	Proficient 4	Advanced 5
I value the need for using appropriate materials, for linking the curriculum to the students' prior learning, and for using an interdisciplinary approach and methods central to the discipline.					
I value the development of students' critical thinking, independent problem solving and performance capabilities.					
I value planning as a short and long term collegial activity open to revision.					
I value the need for assessment of students' learning, and regularly use assessment evidence in making decisions in my instruction.					

The open-ended narratives generated information about participants' views of mathematics teaching and learning as it relates to specific student demographic groups. The nature of the questions reflected the broad propositions of culturally relevant pedagogy. Participants provided a written response to the following open-ended questions:

(a) Education statistics indicate a disparity in the mathematics achievement of Black, Hispanic, and White students in U.S. schools. Why do you believe this disparity exists?

(b) Education statistics also indicate a disparity in the mathematics achievement of low income and middle/high income students in U.S. schools. Why do you believe this disparity exists?

(c) Some educators suggest that low income students better learn mathematics in an environment that is teacher-centered, emphasizes direct instruction, procedural understanding, and seatwork. Do you agree or disagree with this position? Why?

(d) Some educators also suggest that low income and minority students do not possess significant mathematics cultural capital. That is, they do not bring valuable out-of-school experiences and mathematics informal knowledge to the teaching and learning process. Do you agree or disagree with this position? Why?

Participants' mathematics instructional practices were documented via a teaching observation rubric during their student teaching experience. Participants were observed six times over a period of one semester. The indicators on the teaching observation rubric are based on the INTASC standards. See table 2.

Table 2. Field-Based Teaching Observation Rubric

INTASC STANDARDS	PROFICIENT	DEVELOPING	BASIC	UNSATISFACTORY
Standard 1: Knowledge of Subject Matter Standard 7: Instructional Planning Skills	Teacher candidate displays extensive content knowledge and makes connections between the content and other parts of the discipline and other disciplines. Instruction effectively promotes student	Teacher candidate displays solid content knowledge and makes some connections between the content and other parts of the discipline and other disciplines. Instruction moderately promotes student	Teacher candidate displays general content knowledge but cannot articulate connections with other parts of the discipline or with other disciplines. Instruction minimally promotes student	Teacher candidate makes content errors or does not correct content errors students make. Instruction does not promote student conceptual understanding. Learning activities and/or instructional materials are not suitable to students

	conceptual understanding. Learning activities and/or instructional materials are highly relevant to students and instructional goals. They progress coherently, producing a unified whole and reflect best practices.	conceptual understanding. Most of the learning activities and/or instructional materials are suitable to students and instructional goals. Most activities reflect best practices.	conceptual understanding. Some of the learning activities and/or instructional materials are suitable to students and instructional goals. Some activities reflect best practices.	and instructional goals. Activities do not reflect best practices.
Standard 2: Knowledge of Human Development and Learning Standard 3: Adapting Instruction for Individual Needs Standard 4: Multiple Instructional Strategies	All instructional goals and activities are developmentally appropriate. Pacing is appropriate throughout the lesson. Teacher candidate displays thorough knowledge of students' skills and needs and utilizes a variety of instructional strategies, manipulatives, and resources. Instructional practices and interactions consistently reflect high expectations for all learners. Candidate consistently and effectively scaffolds student learning, encourages students to articulate their thinking, probes incorrect or off target responses, and uses errors as a springboard for learning.	Most instructional goals and activities are developmentally appropriate. Pacing is appropriate during most of the lesson. Teacher candidate displays solid knowledge of students' skills and needs and utilizes some variety in instructional strategies, manipulatives, and resources. Instructional practices and interactions moderately reflect high expectations for all learners. Candidate, in most instances, scaffolds student learning, encourages students to articulate their thinking, probes incorrect or off target responses, and uses errors as a springboard for learning.	Only some instructional goals and activities are developmentally appropriate. Pacing is appropriate during some of the lesson. Teacher candidate displays general knowledge of students' skills and needs and utilizes minimal variety in instructional strategies, manipulatives, and resources. Instructional practices and interactions minimally reflect high expectations for all learners. Candidate, in some instances, scaffolds student learning, encourages students to articulate their thinking, probes incorrect or off target responses, and uses errors as a springboard for learning.	Instructional goals and activities are not developmentally appropriate. Pacing is inappropriate. Teacher candidate displays minimal knowledge of students' skills and needs and does not utilize variety in instructional strategies, manipulatives, and resources. Instructional practices and interactions do not reflect high expectations for all learners. Candidate does not scaffold student learning, encourages students to articulate their thinking, probes incorrect or off target responses, and uses errors as a springboard for learning.

Standard 5: Classroom Motivation and Management	Management system and response to student misbehavior are highly effective and consistent. Transitions, routines, and classrooms procedures are well established and highly effective. Strategies to ensure all students are actively engaged are well established and utilized consistently. All learners are on task during the lesson.	Management system and response to student misbehavior are effective and consistent most of the time. Transitions, routines, and classroom procedures are established and effective in most instances. Strategies to ensure all students are actively engaged are moderately established and utilized most of the time. Most learners are on task during the lesson.	Management system and response to student misbehavior are moderately effective and consistent some of the time. Transitions, routines, and classroom procedures are effective in some instances. Strategies to ensure all students are actively engaged are minimally established and utilized some of the time. Only some learners are on task during the lesson.	Management system and response to student misbehavior are ineffective and inconsistent. Transitions, routines, and classroom procedures are not established and ineffective. Strategies to ensure all students are actively engaged are not established and utilized. Learners are not on task during the lesson.
Standard 6:	Teacher candidate	Teacher candidate	Teacher candidate	Teacher candidate

Communication Skills	consistently poses high level questions that promote student thinking. Teacher candidates' dialogue and interactions consistently demonstrate high expectations, genuine caring, and respect. Both verbal and non-verbal communication is always positive.	moderately poses high level questions that promote student thinking. Teacher candidates' dialogue and interactions moderately demonstrate high expectations, genuine caring, and respect. Verbal and non-verbal communication is generally positive.	minimally poses high level questions that promote student thinking. Teacher candidates' dialogue and interactions minimally demonstrate high expectations, genuine caring, and respect. Verbal and non-verbal communication is somewhat positive.	moderately poses high level questions that promote student thinking. Teacher candidates' dialogue and interactions do not demonstrate high expectations, genuine caring, and respect. Verbal and non-verbal communication is not positive.
Standard 8: Assessment of Student Learning	Teacher candidate effectively monitors student learning and makes appropriate modifications during the lesson, based on student responses and feedback. Assessment is congruent with instructional goals, and stated in clear and measurable terms.	Teacher candidate moderately monitors student learning and makes some modifications during the lesson, based on student responses and feedback. Assessment is moderately congruent with instructional goals, but could be strengthened.	Teacher candidate minimally monitors student learning and makes few modifications during the lesson, based on student responses and feedback. Assessment is minimally congruent with instructional goals.	Teacher candidate does not monitor student learning or make modifications during the lesson, based on student responses and feedback. Assessment is not congruent with instructional goals.

Findings and Discussion

The broad propositions of culturally relevant pedagogy served as the lens for organizing and analyzing the data sets. Hence, the participants' disposition self ratings, open-ended responses, and instructional snapshots were logged, chunked, and coded along the following themes: conceptions of self and others, social relations, and conceptions of knowledge. Analyses of data sets followed the schema reflected in table 3.

Table 3. Analyses Schema

	Disposition Self-Ratings	Open-Ended Narratives	Instructional Snapshots
	Ratings logged (1 to 5)	Responses chunked and coded	Practices chunked and coded
Conceptions of Self and Others	I value the individual differences among all students and accommodating these differences to fulfill the learning needs of all students. I value human diversity, respect students' varied talents and perspectives, and show sensitivity to community and cultural norms. I value personal reflection in my development.	Education statistics indicate a disparity in the mathematics achievement of Black, Hispanic, and White students in U.S. schools. Why do you believe this disparity exists? Education statistics also indicate a disparity in the mathematics achievement of low income and middle/high income students in U.S. schools. Why do you believe this disparity exists?	INTASC Standard 2: Knowledge of Human Development and Learning INTASC Standard 3: Adapting Instruction for Individual Needs INTASC Standard 4: Multiple Instructional Strategies
Social Relations	I value an environment in which students have clear expectations and in which time and space in the classroom are managed to ensure student engagement.	Some educators suggest that low income students better learn mathematics in an environment that is teacher-	INTASC Standard 5: Classroom Motivation and Management

	I value the many ways in which people seek to communicate and values responsive listening. I value relationships with school colleagues, parents, agencies, and the community.	centered, emphasizes direct instruction, procedural understanding, and seatwork. Do you agree or disagree with this position? Why?	INTASC Standard 6: Communication Skills INTASC Standard 10: Partnerships
Conceptions of Knowledge	I value the need for using appropriate materials, for linking the curriculum to the students' prior learning, and for using an interdisciplinary approach and methods central to the discipline. I value the development of students' critical thinking, independent problem solving and performance capabilities. I value planning as a short and long term collegial activity open to revision. I value the need for assessment of students' learning, and regularly use assessment evidence in making decisions in my instruction.	Some educators also suggest that low income and minority students do not possess significant mathematics cultural capital. That is, they do not bring valuable out-of-school experiences and mathematics informal knowledge to the teaching and learning process. Do you agree or disagree with this position? Why?	INTASC Standard 1: Knowledge of Subject Matter INTASC Standard 7: Instructional Planning Skills INTASC Standard 8: Assessment of Student Learning

Findings indicate that nine participants rated themselves as advanced or proficient on 10 out of 10 dispositions items. The remaining five participants rated themselves as developing or basic on at least 8 out of 10 disposition items. For the purpose of discussing patterns across the data sets the aforementioned nine participants were grouped together and labeled as the green cluster. Similarly, the other five participants were grouped together and labeled as the yellow cluster.

Cross analyses of participants' disposition self-ratings, open-ended narratives and instructional snapshots indicates that a connection exists between self-ratings and field-based practices. Preservice teachers reporting high levels of dispositional competency (green cluster) tended to assert equity-oriented views about mathematics teaching and learning in their narratives. This also held true for individuals reporting moderate levels of dispositional competency (yellow cluster). Overall, both groups' narratives reflected perspectives congruent with the broad propositions of culturally relevant pedagogy: conceptions of self and others, social relations, and conceptions of knowledge.

Differences between the two clusters were revealed, however, with the instructional snapshots. Along the propositions conceptions of self and others and social relations slight variations in mathematics instructional practices existed between the green and yellow clusters. Though the practices of the groups were similar, individuals in the green cluster netted slightly more instances of practices that were coded as culturally relevant. Within the proposition conceptions of knowledge, however, the variation was more pronounced. Participants reporting high levels of dispositional competency (green cluster) yielded twice the number of instances of practices that were coded culturally relevant as did individuals reporting moderate dispositional competency (yellow cluster). For example, participants in the yellow cluster consistently received positive feedback regarding the extent to which they used frames of reference salient to students within the mathematics teaching and learning process. Similarly, they were more adept at using students' out-of-school experiences to make mathematical connections and promote conceptual understanding.

These findings indeed suggest that there is a connection between teachers' dispositions self-appraisals and their mathematics instructional practices. Particularly interesting is the noted distinction between the different components of culturally relevant pedagogy. It is striking that individuals with differing levels of dispositional competency compare similarly along the propositions conceptions of self and others and social relations, but vary significantly with respect to conceptions of knowledge. This distinction is interesting and worthy of further exploration in a subsequent study. This study reemphasis' the importance of mathematics teacher educators creating disequilibrium in their mathematics methods courses, assignments, and field experiences --- disequilibrium that challenges preservice teachers to critically examine their views of mathematics teaching and learning as it relates to students of color, students living in poverty, and students for whom English is a second language. Further, it illuminates the perspective that good intentions are simply not enough. While caring and affirming views are certainly requisites of a culturally relevant teacher, those affective attributes must be coupled with excellence-oriented, culturally relevant instructional practices.

For equity to be achieved, the mathematics education community as an entirety must be committed to the mathematics excellence of *all* students. That commitment must extend beyond rhetoric and slogans to action which facilitates the accomplishment of that in which we say we believe. In essence, do we have the will?

References

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